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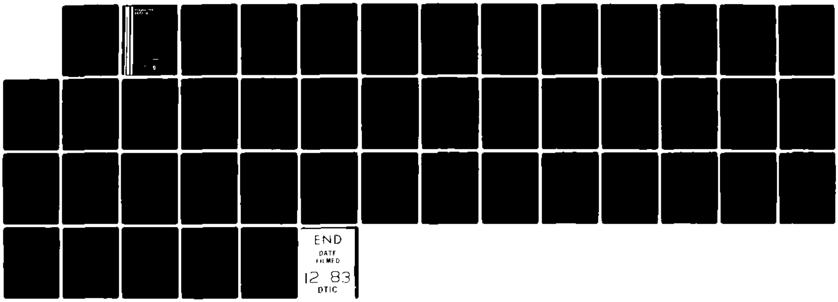

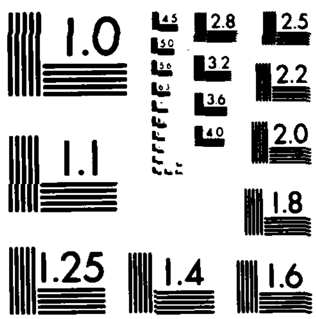

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August 1983

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INTERACTION OF ELECTROMAGNETIC FIELDS WITH PLASMA

by

B.R. Cheo, S.P. Kuo and B.R. Poole

FINAL TECHNICAL PROGRESS REPORT

Prepared For

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

Grant No. AFOSR-79-0009
and
[redacted]

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 83-0879	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Interaction of Electromagnetic Fields With Plasma		5. TYPE OF REPORT & PERIOD COVERED Final Report Oct. 1, 1978-Dec. 31, 1982
		6. PERFORMING ORG. REPORT NUMBER POLY-EE-83-003
7. AUTHOR(s) B.R. Cheo, S.P. Kuo, B.R. Poole		8. CONTRACT OR GRANT NUMBER(s) AFOSR-79-0009
9. PERFORMING ORGANIZATION NAME AND ADDRESS Polytechnic Institute of New York, Electrical Engineering Department, Route 110, Farmingdale, N.Y. 11735		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 2301/A8 61102F
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NP, Washington, D.C.		12. REPORT DATE August, 1983
		13. NUMBER OF PAGES 42
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Essentially five topics have been studied during this time period; three are essentially experimental investigations, and the other two are theoretical in nature. The five topic include: 1) Propagation of microwaves along a plasma column and harmonic generation of electrostatic ion cyclotron waves, 2) Imploding tube experiment, 3) RF Generated Current in a Magnetized Plasma Using a Slow Wave Structure. 4) Wave-particle interaction at cyclotron resonances, and 5) Turbulent interaction between waves and charged particles.

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FINAL TECHNICAL PROGRESS REPORT

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

INTERACTION OF ELECTROMAGNETIC FIELDS WITH PLASMA

(Grants: AFOSR-79-0009, AFOSR-79-0009B)

Period Starting: October 1, 1978

Period Ending : December 31, 1982

Bernard R. Cheo, Principal Investigator

Spencer P. Kuo, Co-Principal Investigator

Brian R. Poole, Research Assistant

Report No. POLY-EE-83-003

POLYTECHNIC INSTITUTE OF NEW YORK

I. INTRODUCTION

Beginning October 1, 1978 a research grant (AF-AFOSR-79-0009) was awarded to the Polytechnic Institute of New York by AFOSR with Professor B.R. Cheo as the Principal Investigator. Since October 1, 1979, Professor S.P. Kuo shared responsibilities as the Co-Principal Investigator and became Principal Investigator in October 1982. Professor B.R. Cheo then assumed responsibilities as Director of the Laboratory of Plasma Engineering and Science as well as Co-Principal Investigator on the subsequent separate OSR program in mm wave generation beginning October 1, 1982. Essentially five topics ~~have been~~ were studied during this time period; three are essentially experimental investigations, and the other two are theoretical in nature. The five topics include:

1) Propagation of microwaves along a plasma column and harmonic generation of electrostatic ion cyclotron waves,

2) Imploding tube experiment,

3) RF Generated Current in a Magnetized Plasma Using a Slow Wave Structure,

4) Wave-particle interaction at cyclotron resonances, and

5) Turbulent interaction between waves and charged particles.

Topics 1-3 comprise the experimental investigation and the last two are theoretical. In the next section we summarize the achievements made on each of these topics.

II. PROGRESS REPORT

1) Harmonic Generation: Two independent investigations have been conducted in Polytechnic Hollow Cathode Discharge plasma device. In the first investigation the phenomena of harmonic generation of the electrostatic ion-cyclotron wave and suppression of the second and enhancement of the third harmonic wave have been observed and a theoretical model to explain it is proposed. The agreement between the two is favorable.

In the experiment the electrostatic ion-cyclotron wave is excited parametrically with the second harmonic electron-cyclotron wave by injecting a monochromatic electromagnetic wave at $f_o = 9.23$ Ghz. The generation of the second and of the third harmonic waves is clearly observed on the spectrum analyser. The evolution process of suppression of the second harmonic and enhancement of the third harmonic wave is studied by varying a d.c. voltage from zero value on the probe that is used to pick up the signal.

Starting with the Maxwell's equations for electrostatic wave we obtained the rate equations for the second and the third harmonics. The nonlinear coupling coefficients can then be calculated by using the Vlasov equation. After a lengthy manipulation we examine the steady state solutions of the rate equations which are compared favorably with the experiments.

In the second investigation, by adjusting the operating parameters of the device, two distinct modes of plasma column can be obtained. The first one is high density ($\omega_{pe} < \omega_o$) with bigger radius and a brighter periphery than the interior. The second one is of low density ($\omega_{pe} < \omega_o$) with a small radius and the apparent light intensity is maximum at the center of the column. The microwave intensity is maximum on the surface of the plasma column for the first mode. The microwave intensity is maximum at the center of the column for the second mode.

A simple theoretical model has been proposed. The plasma is considered as cylindrical dielectric wave medium which is situated in a longitudinally applied d.c. magnetic field. Full Maxwell's equations have been used to examine the problem instead of the usual simplified electrostatic approximation. Agreement between theory and observations is favorable for both modes.

Project is completed. A technical report is issued: (H.M. Huang and B.R. Cheo, Propagation and Nonlinear Studies of Electromagnetic Wave in a Self-Sustained Plamsa Column," Scientific Report No. POLY-EE-80-005, June, 1980).

2) Imploding Tube Experiment: An investigation of electrically exploding metallic thin shells is conducted experimentally and theoretically. An instability is identified experimentally on the tube prior to vaporization. A theoretical analysis is developed that predicts this instability.

The metallic shells are exploded via a fast capacitor bank. The tube is observed with a modified shadowgraph technique and a long wavelength ($\approx$2.3 cm) instability is observed to dominate at the onset. Theoretical treatment of the problem is derived in part from the Z-pinch instability method of Kruskal and Tuck and the mechanical stress analysis is adapted mainly from the techniques of Onat on thin shelled tubes. These methods are integrated to form a theoretical basis for the instability analysis. The experimental observations and the results of the theoretical analysis compare favorably.

Project is completed. A technical report is issued: (Charles D. Hechtman and B.R. Cheo, "Investigation of Electrically Exploding Thin Shelled Metallic Tubes," Scientific Report No. POLY-EE-80-007, October, 1980).

3) RF Driven Currents: The work in this section has not been reported previously. This experimental investigation has reached conclusion and a scientific report (POLY-EE-83-004) is being prepared and will be issued shortly. This work was motivated by the possible steady-state operation of plasma devices by using unidirectional rf waves to drive dc currents. Experiments were performed in our linear ECRH plasma device which uses a set of phased capacitor plates excited by a 3.25 MHz rf source to excite unidirectional electrostatic ($\omega/k_z \ll c$, $\Omega_i \ll \omega \ll \omega_{pe}$, Ω_e) waves. Fig. 1 shows the basic experimental setup used. The slow wave of structure is located in the region of uniform magnetic field and is located at the low density periphery of the plasma column. Fig.2 shows additional detail of the experimental arrangement. Diagnostics used in the experiment work include:

1) Longitudinally and radially movable Langmuir probes used to determine electron temperature, electron density, and space potential.

2) Radially and azimuthally movable current probes, used to measure radial and azimuthal profiles of current density in the z-direction.

3) Electrostatic Energy Analyzer, used to measure the parallel electron energy distribution function.

4) RF potential probe, used in conjunction with rf interferometer to obtain amplitude and phase characteristics of the rf potential associated with electrostatic waves in the plasma.

It is found experimentally that the electron flow is localized radially and azimuthally to the lower hybrid resonance layer near the outer region of the plasma column. Large modifications in the electron energy distribution function are also found in this region as well as large radial phase variations in the rf potential in this region. These modifications depend strongly on the sign of k_z. Detailed experimental results and their interpretation will be presented in a scientific report (POLY-EE-83-004)

(Ph.D. Dissertation, Brian R. Poole) to be issued in the early Fall of 1983. Results of the experimental investigation were presented at the 1983 IEEE International Conference on Plasma Science.

FIG.1 BASIC EXPERIMENTAL SETUP SHOWING SLOW WAVE STRUCTURE

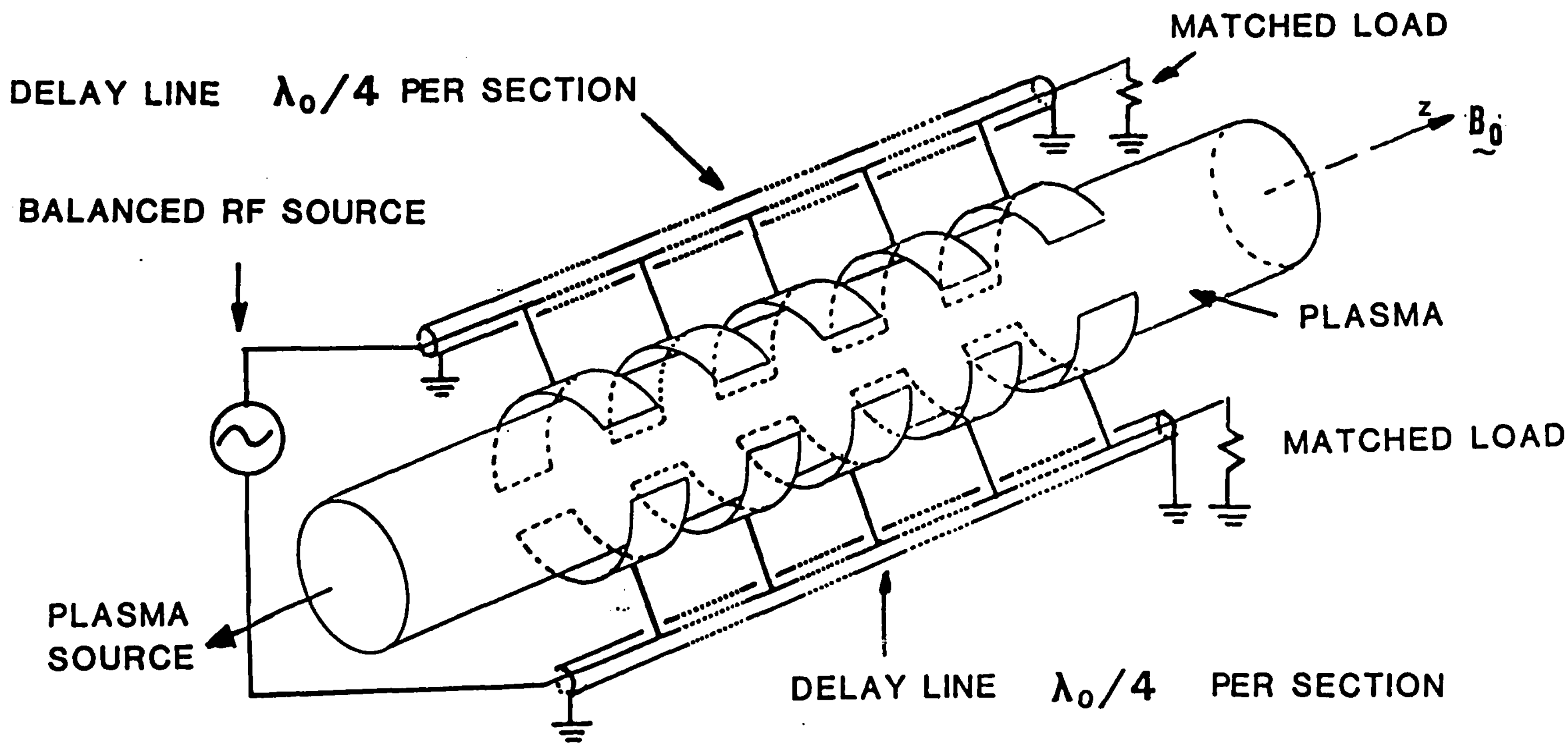
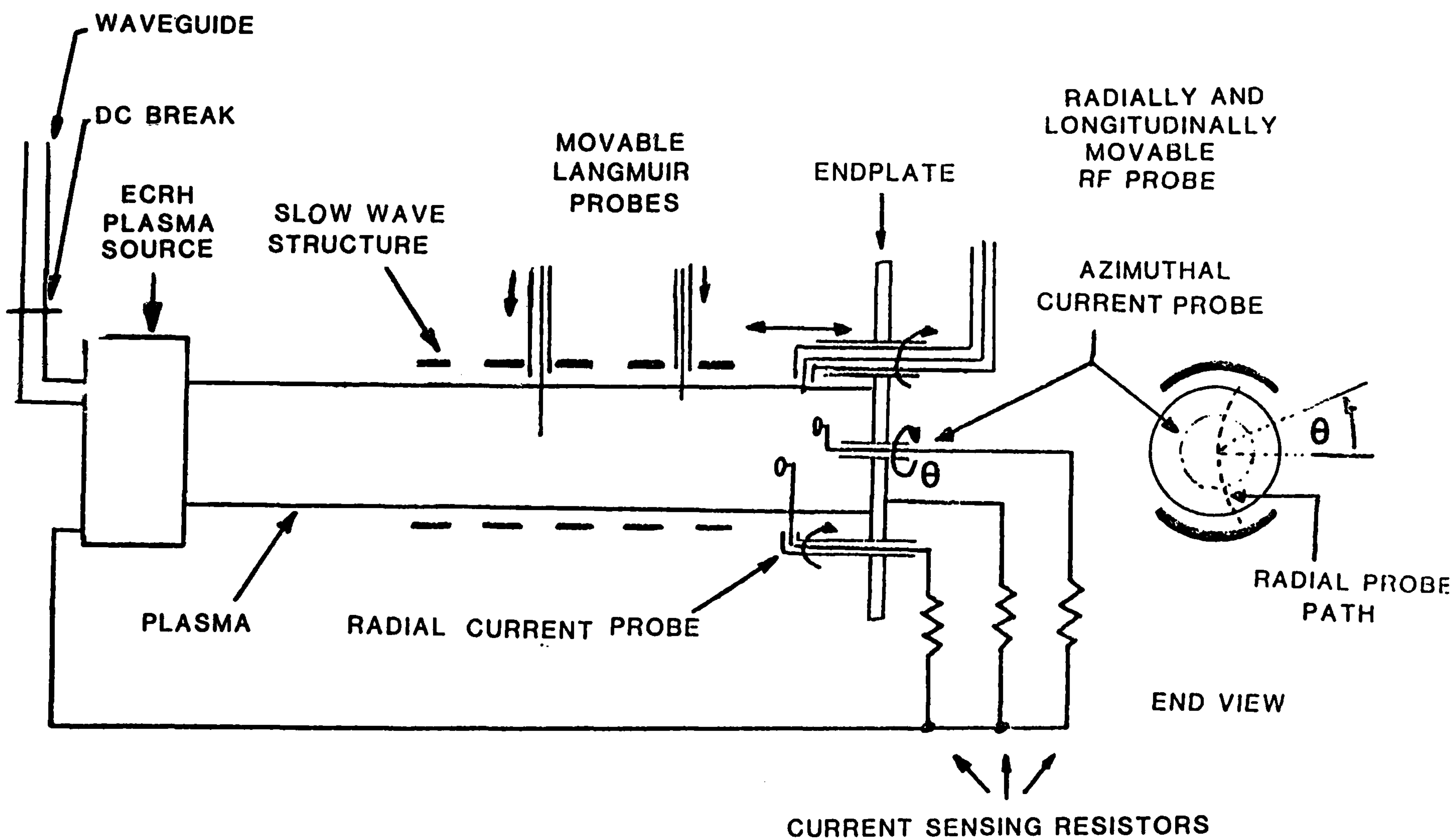


FIG.2 DETAILED EXPERIMENTAL SETUP



4) Wave-Particle Interaction at Cyclotron Resonances:

a) Perpendicular Electron Cyclotron Resonance Heating by Ordinary Modes(2)

Electron heating at resonance by the ordinary mode is analyzed. The equations of motion are integrated along the resonance trajectory showing no parallel heating. Evolution of the distribution function shows that T_1 grows expoentially in t^2 for second harmonic and algebraically at the fundamental.

b) The Effect of the Bouncing Motion of Electrons on Electron Cyclotron Resonance Heating(5)

Cyclotron resonance heating of bouncing electrons by an obliquely incident wave field is analyzed. Continuous interaction between electrons and wave fields throughout the entire range of electron excursion has been considered in the analysis. The results show that the bouncing motion of electrons serves to alleviate the detuning effect of frequency mismatch and efficient heating can be achieved. Kinetic temperature is found to increase algebraically in t^2 for the fundamental resonance and e>_ .nentially in t for the second harmonic heating. Due to the interaction between electrons and the parallel component of the wave electric field, heated electrons tend to focus to the midplane and their excursion amplitudes also become filamented in the steady state.

Copies of these journal articles can be found in the appendix of this report.

5) Turbulent Interaction Between Waves and Charged Particles:

Turbulent Heating of Parametric Instabilities in Unmagnetized Plasmas(3)

Consideration of the effect of a uniform pump field on the particle orbits in a Vlasov plasma leads to a modified diffusion coefficient. When the particles oscillate in the pump field, the turbulent wave

phase velocity seen by the particles is Doppler shifted by multiples of ω_o/k (ω_o is the pump frequency). Hence, strong interactions between the particles and various components of the turbulent field will take place. It is shown that when the pump field is sufficiently strong $E_o \geq (4\pi n_o T_e)^{1/2}$, bulk heating can dominate over tail heating and excitation of electrons to energy levels higher than the normal suprathermal values is possible. This field strength is within the range of laser fusion

III. LIST OF PUBLICATIONS

1) Journal Articles:

1. T. DeNeef and C. Hechtman, "Numerical Study of the Flow Due to a Cylindrical Implosion," Computers and Fluids, Vol. 6, 185-202, 1978.

2. S.P. Kuo and B.R. Cheo, "Perpendicular Electron Cyclotron Resonance Heating by Ordinary Modes," Phys. Fluids, 24, 784-785, 1981.

3. S.P. Kuo and B.R. Cheo, "Turbulent Heating of Parametric Instabilities in Unmagnetized Plasmas," Phys. Fluids, 24, 1104-11[illegible], 1981.

4. T.Q. Yip, S.P. Kuo and B.R. Cheo, "Temporal Evolution of Parametrically Excited Instabilities in a Magnetized Plasma," Plasma Physics, 21, 487-508, 1981.

5. S.P. Kuo and B.R. Cheo, "The Effect of the Bouncing Motion of Electrons on Electron Cyclotron Resonance Heating," accepted for publication in The Physics of Fluids.

2) Conference Presentations:

1. T.Q. Yip, S.P. Kuo and B.R. Cheo, "Evolution of Parametrically Excited Instabilities in a Magneto Plasma," Bull. Amer. Phys. Soc. Vol. 23, 7, 797, 1978.

2. H.M. Huang, S.P. Kuo and B.R. Cheo, "Harmonic Generation of the Electrostatic Ion Cyclotron Wave in a Uniform Magneto-Plasma," Bull. Amer. Phys. Soc., Vol. 23, 7, 818, 1978.

3. S.P. Kuo and B.R. Cheo, "Stabilization of Parametric Decay Instabilities by Heating Effect and Related Nonlinear Phenomena," IEEE Plasma Science International Conference, p. 76, June 4-6, 1979, Montreal, Canada.

4. S.P. Kuo, B.R. Cheo and T.Q. Yip, "Turbulent Bulk Heating of Plasma by Parametric Instabilities," IEEE Plasma Science International Conference, p, 76, 1979, Montreal, Canada

5. S.P. Kuo, B.R. Cheo and E. Levi, "A Transformation for Strong Plasma Turbulence Analysis," Bull. Amer. Phys. Soc., Vol. 24, 8, 851, 1979.

2) Conference Presentations: (continued)

6. B.R. Cheo, S.P. Kuo and E. Levi, "Turbulent Heating of Parametric Instabilities," Bull. Amer. Phys. Soc., Vol. 24, 8, 1015, 1979.

7. S.P. Kuo and B.R. Cheo, "Perpendicular ECRH of EM Waves," Bull. Amer. Phys. Soc., Vol. 22, 8, 1020, 1980.

8. S.P. Kuo, B.R. Cheo and D. Wu," Heating Rate Equations of Charged Particles by RF Fields at Cyclotron Resonance," IEEE Plasma Science International Conference, 81 CH 1640-2NPS,162-163, 1981.

9. S.P. Kuo and B.R. Cheo, "Electron Cyclotron Resonance Heating in a Mirror Field," IEEE Plasma Science International Conference, 81 CH 1640-2NPS, 162, 1981.

10. B.R. Cheo and S.P. Kuo, "Enhanced Trapping of Electrons by the Cyclotron Resonance in a Bouncing Field," IEEE Plasma Science International Conference, 82 CH 1774-7, 36, 1982.

11. S.P. Kuo, "Effects of an Axial Guide Field on the Operations of Free Electron Lasers," IEEE Plasma Science International Conference, 82 CH 1770-7, 181, 1982.

12. S.P. Kuo, "Ponderomotive Force Near the Cyclotron Harmonic Resonances," IEEE Plasma Science International Conference, 82 CH 1770-7, 26, 1982.

13. B.R. Poole, B.R. Cheo, S.P. Kuo and M.G. Tang, "RF Generated Currents in a Magnetized Plasma Using a Slow Wave Structure," IEEE International Conference on Plasma Science, 83 CH 1847-3, 97, 1983.

3) Technical Reports:

1. C.D. Hechtman and B.R. Cheo, "Investigation of Electronically Exploding Thin Shelled Metallic Tubes," Scientific Report No. POLY-EE-80-007, October, 1980.

2. H.M. Huang and B.R. Cheo, "Propagation and Nonlinear Studies of EM-Waves in a Self-Sustained Plasma Column," POLY-EE-80-005, June, 1980.

3. B.R. Poole and B.R. Cheo, "RF Generated Currents in a Magnetized Plasma Using a Slow Wave Structure," POLY-EE-83-004, to be completed.

IV. APPENDIX

This section contains important journal publications relating to the work on cyclotron resonance and turbulence.

Perpendicular electron cyclotron resonance heating by ordinary modes

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(Received 6 May 1980; accepted 8 January 1981)

Electron heating at resonance by the ordinary mode is analyzed. The equations of motion are integrated along the resonance trajectory showing no parallel heating. Evolution of the distribution functions shows that $T_\perp$ grows exponentially in t^2 for second harmonic heating and algebraically at the fundamental.

Recently, there has been renewed interest in electron cyclotron resonance heating[1-7] of plasmas in connection with the need for electron heating in magnetically confined devices, and with ionospheric heating experiments. It plays a crucial role in the operation of the Elmo Bumpy Torus.[8,9] It was thought that because of the parallel polarization of the electric field of the heating wave, ordinary modes would only lead to parallel heating. However, in the following, we shall show that at resonance, when the effect of the oscillating magnetic field is included, there is strong perpendicular heating and no parallel heating at all.

We follow standard procedures to determine particle motion in the heating wave field of an ordinary mode including the effect of the oscillating magnetic field of the heating wave in the Lorentz force. A perturbed (self-consistent) trajectory is found and used in the integration of the equations of motion at resonance ($\omega = n\Omega$). It is found that the parallel heating term vanishes exactly at resonance in agreement with previous work[10,11] using a first-order quasi-linear diffusion theory (weak heating), but differs from the result of single particle theory[2] which neglected the oscillating magnetic field. Efficient perpendicular heating is predicted. The temporal evolution of the electron distribution function and of the perpendicular temperature are derived for the fundamental resonance and for the second harmonic.

In a homogeneous plasma background imbedded in a uniform field $\mathbf{B}_0$, and an electromagnetic field having ordinary polarization, the electron motion obeys:

$$\frac{d\mathbf{r}(t)}{dt} = \mathbf{v}(t), \tag{1}$$

$$\frac{d\mathbf{v}(t)}{dt} = -\left(\frac{e}{m}\right)\left(\mathbf{E}[\mathbf{r}(t), t] + \frac{\mathbf{v}(t)}{c}\times\{\mathbf{B}[\mathbf{r}(t), t] + \mathbf{B}_0\}\right), \tag{2}$$

where $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ are oscillating fields of the wave. We choose a coordinate system in which

$$\mathbf{B}_0 = \hat{z}B_0, \quad \mathbf{k} = \hat{x}k, \quad \mathbf{E}(\mathbf{r}, t) = \hat{z}E_1\cos(kx - \omega t),$$

and $\mathbf{B}(\mathbf{r}, t) = -\hat{y}B_1\cos(kx - \omega t)$, where $B_1 = kcE_1/\omega$. Let $v^- = v_x - iv_y$, hence, (1) and (2) can be integrated as

$$\mathbf{r}(t) = \mathbf{r}_0 + \frac{\mathsf{H}(t)\cdot\mathbf{v}_0}{\Omega} - \frac{c}{B_0}\int_0^t ds\, \mathsf{H}(t-s)\cdot\left(\mathbf{E}[\mathbf{r}(s), s] + \frac{\mathbf{v}(s)}{c}\times\mathbf{B}[\mathbf{r}(s), s]\right), \tag{3}$$

$$v^-(t) = v_{\perp 0}\exp[-i(\Omega t + \theta_0)] - \frac{\Omega B_1}{B_0}\exp(-i\Omega t) \times \int_0^t ds\, v_z(s)\exp(i\Omega s)\cos[kx(s) - \omega s], \tag{4}$$

$$v_z(t) = v_{z0} + \frac{\Omega B_1}{B_0}\int_0^t ds\left(v_x(s) - \frac{cE_1}{B_1}\right) \times \cos[kx(s) - \omega s], \tag{5}$$

where $\Omega = eB_0/mc$ and the initial conditions $\mathbf{r}_0 = \mathbf{r}(0)$,

$$\mathbf{v}_0 = \mathbf{v}(0) = v_{\perp 0}(\hat{x}\cos\theta_0 + \hat{y}\sin\theta_0) + \hat{z}v_{\parallel}(0),$$

and

$$\mathsf{H}(t) = (\hat{x}\hat{x} + \hat{y}\hat{y})\sin\Omega t - (\hat{x}\hat{y} - yx)(1 - \cos\Omega t) + \hat{z}\hat{z}\Omega t.$$

In order to find the exact trajectory, (3)–(5) must be solved simultaneously. However, at resonances $\omega = n\Omega$, the response to the resonance interaction dominates all other terms. The exact trajectory would then be that given by the resonant (secular) term modulated by negligibly small oscillations due to the other terms which are smaller by a factor $(\Omega t)^{-2}$. In this limit, we have:

$$x(t) = x_0 + [w(t)/k]\sin\theta(t) - (v_{\perp 0}/\Omega)\sin\theta_0, \tag{6}$$

where

$$w(t) = [(\eta + k\beta)^2 + (k\alpha)^2]^{1/2}, \quad \theta(t) = \theta_0 + \Omega t + \psi(t), \tag{7}$$

$$\alpha(t) = \frac{B_1 v_{z0}}{2B_0}\int_0^t dt'\{J_{n-1}(w')\sin[\Phi + (n-1)\psi'] - J_{n+1}(w')\sin[\Phi + (n+1)\psi']\}, \tag{8}$$

$$\beta(t) = -\frac{B_1 v_{z0}}{2B_0}\int_0^t dt'\{J_{n-1}(w')\cos[\Phi + (n-1)\psi'] + J_{n+1}(w')\cos[\Phi + (n+1)\psi']\}, \tag{9}$$

with $\Phi = kx_0 - \eta\sin\theta_0 + n\theta_0$, $\eta = kv_0/\Omega$, $w' = w(t')$, $\psi' = \psi(t')$, and $\psi(t) = \tan^{-1}[k\alpha/(\eta + k\beta)]$. Substituting (6) and the resonance response part of $v_x = [\Omega w/k)\cos\theta(t)]$ into (5), yields

$$v_z(t) = v_{z0} + \frac{\Omega B_1}{4B_0}\left[\exp(i\Phi)\int_0^t dt'\left(v_{\perp 0} + \Omega\beta(t') \times \{J_{n+1}(w')\exp[i(n+1)\psi'] + J_{n-1}(w') \times \exp[i(n-1)\psi']\} - \frac{2cE_1}{B_1}J_n(w')\exp(in\psi') - i\Omega\alpha(t')(J_{n+1}(w')\exp[i(n+1)\psi'] - J_{n-1}(w') \times \exp[i(n-1)\psi']\right)\exp[-i(\omega - n\Omega)t'] + \text{c. c.}\right]. \tag{10}$$

If the oscillating magnetic field is neglected and the un-

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perturbed trajectory is used, previous results can be recovered.[2] Using the trajectory (6), Eq. (10) becomes

$$\begin{aligned} v_z(t) = v_{z0} &+ \frac{\Omega B_1}{2B_0} \int_0^t dt' \Big([v_{\perp 0} + \Omega\beta(t')] \\ &\times \{ \cos[\phi + (n+1)\psi'] J_{n+1}(w') \\ &+ \cos[\phi + (n-1)\psi'] J_{n-1}(w') \} - (2n\Omega/k) \cos(\phi + n\psi') \\ &\times J_n(w') + \Omega\alpha(t') \{ \sin[\phi + (n+1)\psi'] J_{n+1}(w') \\ &- \sin[\phi + (n-1)\psi'] J_{n-1}(w') \} \Big) \\ = v_{z0} &+ \frac{\Omega^2 B_1}{2kB_0} \int_0^t dt'\, w' \cos(\phi + n\psi') [J_{n+1}(w') \\ &+ J_{n-1}(w') - (2n/w') J_n(w')] = v_{z0} . \end{aligned} \tag{11}$$

Here, the relations $\cos\psi = (\eta + k\beta)/w$, $\sin\psi = k\alpha/w$, $\omega = n\Omega$, and the identity $J_{n+1}(w) + J_{n-1}(w) - (2n/w)J_n(w) = 0$ have been employed. It is seen that there is *no* parallel resonant heating at all. Physically, this may be explained as follows: When an electron is moving in resonance with the field along the resonant trajectory, it sees an electric field consisting of a constant term plus a series of oscillations at $n\Omega$. The constant term of the field is proportional to $J_n(w)$. In addition, in the z direction the electron also experiences a $\mathbf{v}\times\mathbf{B}$ force consisting of two constant terms plus oscillations. The two constant terms, one upshifted and one downshifted, proportional to $J_{n\pm 1}(w)$, respectively, would exactly cancel that due to the electric field. The net Lorentz force on the electron in the parallel direction is hence oscillatory and does not contribute to heating.

Substituting (11) into (4), we further obtain

$$v^-(t) = [\Omega w(t)/k] \exp[-i\theta(t)], \quad v_\perp^2(t) = (\Omega w/k)^2 . \tag{12}$$

With the aid of (8) and (9) and the trajectories (6), (7), (11), and (12), the temporal evolution of the perpendicular energy of electrons and the electron distribution function can be determined. In the collisionless limit, the evolution of the distribution function follows the equation $df_e(\mathbf{v}, \mathbf{r}, t)/dt = 0$ along the trajectories (6), (7), (11), and (12). This equation gives the relation $f_e(\mathbf{v}, \mathbf{r}, t) = f_e(\mathbf{v}_0, \mathbf{r}_0, 0)$ provided that $(\mathbf{v}, \mathbf{r})$ are related to $(\mathbf{v}_0, \mathbf{r}_0)$ through the relations (6), (7), (11), and (12). Thus, for a given initial distribution $f(\mathbf{v}_0, \mathbf{r}_0)$, we may determine the time evolved distribution function under the resonance heating limit. The kinetic perpendicular temperature $T_\perp(t)$ is thus:

$$T_\perp(t) = \frac{m}{2} \int \{[v_{\perp 0} + \Omega\beta(t)]^2 + [\Omega\alpha(t)]^2\} f_e(\mathbf{v}_0, \mathbf{r}_0, 0) d\mathbf{v}_0 . \tag{13}$$

We consider two cases for an initial Maxwellian $f_e(\mathbf{v}_0, \mathbf{r}_0, 0) = (m/2\pi T_0)^{3/2} \exp[-(m/2T_0)(v_{\perp 0}^2 + v_{z0}^2)]$:

(i) Fundamental cyclotron resonance heating ($n = 1$): Let $J_0(w) \approx 1$ and $J_2(w) \approx 0$ for $|w| \ll 1$, $(w \cong v_\perp/c)$, thus $\alpha(t) \approx (B_1 v_{z0} t/2B_0) \sin\phi$ and $\beta(t) \approx -(B_1 v_{z0} t/2B_0) \cos\phi$. Since $v_{z0} = v_z$, $v_{\perp 0}^2 = v_\perp^2 + (B_1 v_z \Omega t/2B_0)^2 \cos 2\Phi + (B_1 v_z \Omega t/B_0) \times \cos\Phi (v_\perp^2 - (B_1 v_z \Omega t/2B_0)^2 \sin^2\Phi)^{1/2}$, and $\theta_0 = \theta - \Omega t - \psi = \theta - \Omega t$, and thus $x_0 - (v_{\perp 0}/\Omega)\sin\theta_0 = x - (v_\perp/\Omega)\sin\theta$, $\Phi = k[x - (v_\perp/\Omega)\sin\theta] + (\theta - \Omega t)$, we then obtain

$$\begin{aligned} f_e(\mathbf{v}, \mathbf{r}, t) &= f_e(v_\perp, \theta, v_z, x, t) \\ &= (m/2\pi T_0)^{3/2} \exp(-(m/2T_0)\{v_z^2 + v_\perp^2 \\ &\quad + (B_1 v_z \Omega t/2B_0)^2 \cos 2\phi + (B_1 v_z \Omega t/B_0) \\ &\quad \times \cos\phi [v_\perp^2 - (B_1 v_z \Omega t/2B_0)^2 \sin^2\phi]^{1/2}\}) \end{aligned} \tag{14}$$

$$T_\perp(t) = T_0[1 + (ekE_1/m\omega)^2 t^2/8] . \tag{15}$$

(ii) Second harmonic cyclotron resonance heating ($n = 2$): We keep only the J_1 term in (9) and (10) and let $J_1(w) \approx w/2$ for $|w| < 1$, $(w \cong 2v_\perp/c)$. Since $\cos\phi = (\eta\, k\beta)/w$, and $\sin\psi = k\alpha/w$, we can then solve (9) and (10) and obtain $\eta + k\beta(t) = \eta[\cosh(kat) - \cos\phi \sinh(kat)]$ and $k\alpha(t) = \eta \sin\phi \sinh(kat)$, where $a = B_1 v_{z0}/4B_0$. We now have $v_{z0} = v_z$, $v_{\perp 0}^2 = v_\perp^2[\cosh(2kat) - \cos\Phi \sinh(2kat)]^{-1}$, $a = B_1 v_z/4B_0$ and $\Phi = k[x - (v_\perp/\Omega)\sin\theta] + 2(\theta - \Omega t)$. Thus, we have

$$\begin{aligned} f_e(\mathbf{v}, \mathbf{r}, t) &= f_e(v_\perp, \theta, v_z, x, t) = (m/2\pi T_0)^{3/2} \\ &\quad \times \exp(-(m/2T_0)\{v_z^2 + v_\perp^2[\cosh(2kat) \\ &\quad - \cos\phi \sinh(2kat)]^{-1}\}) , \end{aligned} \tag{16}$$

$$T_\perp(t) = T_0 \exp[(T_0/2m)(k^2 eE_1/m\omega^2)^2 t^2] . \tag{17}$$

The results show that for long interaction times, the second harmonic heating, which increases exponentially in t^2, is more efficient than the fundamental. Note that equation (15) is different from that given in Ref. 11 where $T_\perp$ increases as t.

This work was supported by the Air Force Office of Scientific Research, Grant No. AFOSR-79-7009.

[1] R. A. Dandl, A. C. England, W. B. Ard, H. O. Eason, M. C. Becker, and G. M. Haas, Nucl. Fusion 4, 344 (1964); R. A. Dandl, H. O. Eason, P. H. Edmonds, and A. C. England, Nucl. Fusion 11, 411 (1971).
[2] O. Eldridge, Phys. Fluids 15, 676 (1972).
[3] J. C. Sprott, Phys. Fluids 14, 1795 (1971); 15, 2247 (1972).
[4] H. Grawe, Plasma Phys. 11, 151 (1968).
[5] F. Jaeger, A. J. Lichtenberg, and M. A. Lieberman, Plasma Phys. 14, 1973 (1972).
[6] M. A. Lieberman and A. J. Lichtenberg, Plasma Phys. 15, 125 (1973).
[7] J. D. Barter, J. C. Sprott, and K. L. Wong, Phys. Fluids 17, 810 (1974).
[8] R. A. Dandl, H. O. Eason, and H. Ikegami, Oak Ridge National Laboratory Report, TM-6703 (1979).
[9] R. A. Dandl (private communication).
[10] V. L. Yakimenko, Zh. Eksp. Teor. Fiz. 44, 1534 (1963) [Sov. Phys.–JETP 17, 1032 (1973)].
[11] J. Ravlands, V. L. Sizenenko, and K. N. Stepanov, Zh. Eksp. Teor. Fiz. 50, 994 (1966). [Sov. Phys.–JETP 23, 661 (1966)].

THE EFFECT OF THE BOUNCING MOTION OF ELECTRONS ON ELECTRON CYCLOTRON RESONANCE HEATING

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ABSTRACT

Cyclotron resonance heating of bouncing electrons by an obliquely incident wave field is analyzed. Continuous interaction between electrons and wave fields throughout the entire range of electron excursion has been considered in the analysis. The results show that the bouncing motion of electrons serves to alleviate the detuning effect of frequency mismatch and efficient heating can be achieved. Kinetic temperature is found to increase algebraically in t^2 for the fundamental resonance and exponentially in t for the second harmonic heating. Due to the interaction between electrons and the parallel component of the wave electric field, heated electrons tend to focus to the midplane and their excursion amplitudes also become filamented in the steady state.

I. Introduction

Interactions between high frequency electromagnetic fields and plasmas have been studied extensively in connection with wave propagation in the ionosphere and with the studies of plasma heating with high-power microwaves. The commonly recognized mechanism for achieving effective electron heating is the electron cyclotron resonance heating (ECRH). Recently, ECRH has been adapted to toroidal devices and received considerable attention due to the encouraging results reported for experiments in the Elmo Bumpy Torus (EBT)[1,2] and in Tokamaks[3,4].

Several theories have been developed to explain the observed heating rates. Usually the problem can be approached by two different ways. One approach is to study the wave propagation in plane-stratified plasma. Techniques of geometrical optics[5] have been employed for this purpose, and it was found that full wave theory is required near the resonant surfaces[6,7]. Another approach is by solving the equations of motion of a single electron moving in the wave fields[8-15] and averaging the results over an initial distribution. However, for studying the localized heating phenomenon, for instance the heating of electron rings in the EBT device, the latter approach may give better insight of the physics. It is important to note that the heating efficiency depends on the rate of net energy gain throughout the entire period of particle-wave interaction. To make this point clear, we recall that the power delivered from the wave to the charged particle is the dot product of the electric field force of the wave acting on the charge particle and the velocity of the charge particle. Even if the force and the velocity are initially large and in the same direction (so giving large initial heating rate), if the phase angle difference between these two vector increases linearly in time, the dot product will oscillate becoming zero on the average. Thus, the heating efficiency will be zero on the average. This is why resonance heating is effective, because at exact resonance the phase angle between heating force and particle velocity remains constant (fundamental resonance case) in the linear region and continuous energy flow from wave to charge particle (or vice versa can

be achieved. However, an exact resonance condition is impractical in reality, and a small mismatch to the resonance condition means a strong deterioration in the "effective" heating time and so in heating efficient, the detuning effect.

There are two interlinked sources of detuning: one is the inhomogenity of the background magnetic field and the other is the motion of the particle guiding center.

In a uniform magnetic field case, for instance, if the heating wave is not exactly normal incidence (i.e. $k_{11} \neq 0$) and also if there is no other imposed force field in the system, the $k_{11}z$ phase angle in the wave fields can introduce a detuning effect of frequency mismatch to the charged particles and the continuous heating scheme breaks down. Unless other effects such as collisions and stochasticity[14,15] are set into the system to randomize the detuning phase, effective heating can not be achieved.

In the analyses of such cases one normally assumes that a single particle will repeatedly pass through a narrow resonance zone and gaining a finite but small velocity Δv. After passing the resonance zone the particle drifts through a large region of detuning, suffering a large phase shift $\Delta\theta$ without gaining energy. After many such passings through the resonance zone, the scatter of the $\Delta\theta$'s is presumably large, that an isotropic outward diffusion in the $v_\perp$ space, and hence a net gain of the rms value of velocity, will result. Heating is thus achieved.

The purpose of this paper is to present a different view point. We believe that the continuous particle-wave interaction throughout the entire range of the particle excursion is important. Both the magnitude and the phase of the velocity evolve continuously whether the particle is in or out of the resonance zone. Net energy gain can be fundamentally more efficient than that of the stochasticity approach. To bring out the physics of this continuous interaction we choose a model for which a self-consistent analysis is possible. We assume a uniform background field but the particles are confined in the direction parallel to the magnetic

field by an externally imposed field to form a simple harmonic oscillator. Thus the detuning effect appears in the $k_{11}z$ term of the phase function. In Section II we first formulate the problem and derived the coupled equations for the slow varying amplitude and phase of both the transverse and parallel motions. In Section III, the results are analyzed in detail, giving the heating rates and specific features on the parallel bounce motion. In the final section, a discussion on the main features of the physics involved is presented.

II. Heating Rate Equations

The equations of motion of electrons are

$$\frac{d}{dt}\vec{r} = \vec{v} \tag{1}$$

$$\frac{d}{dt}\vec{v} = -\frac{e}{m}[\vec{E} + \frac{1}{c}\vec{v} \times (\vec{B} + \hat{z} B_o)] - \omega_B^2 z \hat{z} \tag{2}$$

where $-\omega_B^2 z$ is a harmonic bouncing force acting on electrons and introduced by an external force field, and $\vec{E}$ and $\vec{B}$ are the transverse wave fields given by

$$\vec{E} = (\hat{z} k_\perp - \hat{x} k_{11})(E/k)\cos(k_\perp x + k_{11} z - \omega t)$$

$$\vec{B} = -\hat{y}(kcE/\omega)\cos(k_\perp x + k_{11} z - \omega t)$$

In order to provide some physical insight into the mathematical procedure employed to study the problem, we first decompose Eq. (2) into transverse and parallel components and write the transverse equation formally in integral form:

$$v_x - i\, v_y = v_{\perp o} e^{-i(\theta_o + \Omega t)} + (\Omega/\omega)(kcE/B_o) e^{-i(\theta_o + \Omega t)} \int_o^t dt' \, e^{i(\theta_o + \Omega t')} [(k_{11}\omega/k^2) - v_z'] \cos(k_\perp x' + k_{11} z' - \omega t') \tag{3}$$

$$\frac{d}{dt} v_z = (\Omega/\omega)(kcE/B_o)[v_x - (k_\perp \omega/k^2)] \cos(k_\perp x + k_{11} z - \omega t) - \omega_B^2 z \tag{4}$$

where $\Omega = eB_o/mc$ and $\theta_o = \tan^{-1}(v_{yo}/v_{xo})$.

The integrand of (3) represents the instantaneous Lorentz force due to the heating wave seen by the gyrating electrons at (x', z'), and at time t'. Because of the gyrating motion, x' is thus an oscillatory function, nearly sinusoidal at frequency Ω, with slowly varying amplitude and phase. Therefore the cosine function in (3) may be formally expanded into an infinite series, each term successfully smaller and oscillating at the Dopler shifted frequency $\omega - m\Omega$. If the heating wave frequency is one of the harmonics of the cyclotron frequency $(\omega = n\Omega)$, the $m = n$ term is the slowest term and dominates over all other terms which, after integration, are smaller by at least a factor $(\Omega t)^{-1}$ and can be neglected. If further the effect of $k_{11}z$ could be neglected, e.g. $k_{11} = 0$, this term would be non-oscillatory (refer to as the secular term). Exact resonance is achieved and heating will result. However, if $k_{11}z$ cannot be ignored such as the case of free streaming particles, the phase $k_{11}v_z t$ gives rise to a fast oscillating force resulting in detuning. The main theme of this paper is to deal with the effect of a bouncing motion (contrasting to free streaming) and to show that, in this case, efficient heating can be achieved. One may employ a similar argument used in dealing with the gyrating motion to deal with a sinusoidal oscillation in z. One can then define a set of self-consistent resonance trajectories having the form

$$
\begin{aligned}
x &= x_o - (v_{\perp o}/\Omega)\sin\theta_o + (v_\perp/\Omega)\sin(\theta_o + \psi + \Omega t) \\
v_x &= v_\perp \cos(\theta_o + \psi + \Omega t) \\
v_y &= v_\perp \sin(\theta_o + \psi + \Omega t) \\
z &= Z(t)\sin(\Phi + \omega_B t)
\end{aligned}
\tag{5}
$$

where ω_B is the bouncing frequency and the slow time varying functions $v_\perp(t)$, $\Psi(t)$, $Z(t)$ and $\Phi(t)$ are governed by the equations, determined in a self-consistent way by substituting (5) back into Eqs. (3) and (4). The details of this derivation are shown in the Appendix.

$$\frac{d}{dt} v_\perp = (k_{11}/k)(eE/m) J_o(k_{11}Z)[nJ_n(k_\perp v_\perp/\Omega)]/(k_\perp v_\perp/\Omega)]\cos(n\psi + \phi_o) \tag{6}$$

$$\frac{d}{dt} \psi = -(k_{11}k_\perp/k)(eE/m\Omega) J_o(k_{11}Z)[J_n'(k_\perp v_\perp/\Omega)/(k_\perp v_\perp/\Omega)]\sin(n\psi + \phi_o) \tag{7}$$

$$\frac{d}{dt} Z = -|C_o|(Z_o^2 - Z^2)^{\frac{1}{2}} \; |J_1(k_{11}Z)|/Z \tag{8}$$

$$\Phi(t) = \sin^{-1}\{[z_o Z + (v_{zo}/\omega_B)(Z_o^2 - Z^2)^{\frac{1}{2}}]/Z_o^2\} \tag{9}$$

with the initial conditions $v_\perp(o) = v_{\perp o}$, $\psi(o) = 0$, $Z_o = (z_o^2 + v_{zo}^2/\omega_B^2)^{1/2}$, and $\Phi(o) = \tan^{-1}(\omega_B z_o/v_{zo})$ respectively, and $C_o = (k_{11}^2/k_\perp k)(cE/B_o)(\Omega/\omega_B) J_n(k_\perp v_{\perp o}/\Omega)\sin\phi_o$ where $\phi_o = k_\perp x_o - (k_\perp v_{\perp o}/\Omega)\sin\theta_o + n\theta_o$, and subscripts "o' represents initial quantities.

III. Analysis

Equations (6) and (7) can be combined to obtain an invariant relation

$$J_n(k_\perp v_\perp/\Omega)\ \sin(n\psi + \phi_o) = I = \text{constant in time} \tag{10}$$

Eq. (10) shows that the result of resonance is to change $v_\perp$ and ψ continuously and simultaneously, following a closed contour in the polar $v_\perp - n\psi$ coordinate space. However, the part of the contour farthest from the origin reaches into the region for $v_\perp$ too large for the relativisitc effect to be neglected. Hence, within the non-relativistic regime, the heating is a continuous process as will be demonstrated and further discussed later. One notices that here in (10) v_z (and thus Z) does not appear explicitly. This is because when the axial excursion becomes bounded due to bouncing, the time varying phase difference "$k_{11}z$" between wave field and the gyration motion of electrons also becomes a bounded function. After the Bessel function expansion, this detuning effect only appears as a factor of $J_o(k_{11}Z)$ in reducing the strength of resonance coupling [for example, see Eq. (6)], but not giving a detuning frequency any more. This modification in the coupling strength, instead of mismatch frequency, will cancel out when taking the

ratio of Eq. (6) to Eq. (7) for deriving the adiabatic invariant relation. Therefore, the resultant invariant relation (10) does not involve v_z; and $v_\perp$ and v_z only couple to each other weakly so that the change rate of $v_\perp$ depends on v_z through $J_o(k_{11}Z)$ factor. Another adiabatic invariant relation for Z can also be derived by Eqs. (6) and (8). Since there is no need to have this relation to obtain the final results, this relation is not presented.

Physically, one can describe the effect of the bouncing motion to the resonance heating in the following way. The power delivered from the wave to the transverse energy of electrons is determined by the dot product of $\vec{v}_\perp$ and the transverse Lorentz force $\vec{F}_\perp$. In the gyrating frame, one may assume $\vec{v}_\perp$ is more or less fixed in one direction but $\vec{F}_\perp$ moves around $\vec{v}_\perp$ due to the time varying phase difference. If the phase difference is a linearly increasing (or decreasing) time function, then $\vec{F}_\perp$ is rotating around $\vec{v}_\perp$ at the detuning frequency. During half period electron is gaining energy but giving the absorbed energy back to the wave during the next half period, and thus the net power delivered from wave to electrons is zero on the average. The total energy gained by electrons is then limited to the fake heating which is inversely proportional to the square of the detuning frequency, a very small value. Without collisions or stochastic effect etc., heating cannot be continuous. However, if electrons are bouncing in the axial direction, thus $k_{11}z$ becomes a bounded function. In other words, $\vec{F}_\perp$ will be vibrating about $\vec{v}_\perp$ instead of gyrating around $\vec{v}_\perp$. Therefore, $\vec{F}_\perp \cdot \vec{v}_\perp$ is non-zero over time averaged. There is a continuous net power flow from wave to electrons or vice versa, depending on the initial conditions. But as shown later electrons will always gain energy continuously when averaged over the initial conditions. Continuous heating is achieved. This is shown by the perpendicular heating rate equation derived from Eqs. (6) and (10) as

$$\frac{d}{dt} v_\perp^2 = 2(k_{11}\omega/k_\perp k)(eE/m)J_o(k_{11}Z)[J_n^2(k_\perp v_\perp/\Omega) - I^2]^{\frac{1}{2}} \tag{11}$$

Equation (11) should be considered simultaneously with Eq. (8) and can be solved numerically. However the excursion amplitude Z(t) evolves very slowly in time, Eq. (11) alone may be used to determine the heating rate corresponding to the initial phase of the heating by letting $Z = Z_o$. If one further recognizes that the argument of the Bessel function $(k_\perp v_\perp/\Omega)$ is $O(v/c)$ and thus $J_n(x) \approx \frac{1}{n!}(\frac{x}{2})^n$, Eq. (11) can then be integrated analytically. For the cases of $n = 1$ (fundamental) and $n = 2$ (second harmonic) we obtain:

For $n = 1$:

$$v_\perp^2 = v_{\perp o}^2 + (eE/m)(k_{11}/k)J_o(k_{11}Z_o)\cos\phi_o\, v_{\perp o}t$$
$$+ (eE/2m)^2(k_{11}/k)^2 J_o^2(k_{11}Z_o)t^2 \qquad (12)$$

For $n = 2$:

$$v_\perp^2 = v_{\perp o}^2\,[\cosh(bt) + \cos\phi_o\,\sinh(bt)] \qquad (13)$$

where $b = (eE/2m\Omega)(k_{11}k_\perp/k)J_o(k_{11}Z_o)$.

The above two equations give the time evolution of the perpendicular velocity $v_\perp^2$ of a typical particle with initial conditions $\vec{v}_{\perp o}$, x_o (imbedded in ϕ_o) and an initial excursion amplitude Z_o.

To obtain the kinetic perpendicular temperature $T_\perp(t)$ (12) and (13) must be averaged over an initial distribution of the particles. In the perpendicular direction the averaging is straight forward since the initial conditions appear only as a phase ϕ_o. With the averaging in the parallel direction formally expressed we have:

n = 1

$$T_\perp(t) = T_{\perp o} + \langle J_o^2(k_{11}Z_o)\rangle (e^2E^2/8m)(k_{11}/k)^2 t^2 \tag{14}$$

and n = 2

$$T_\perp(t) = T_{\perp o}\langle \cosh bt\rangle \tag{15}$$

In the parallel direction, the particles have both kinetic and potential energies (harmonic oscillators). The excursion Z_o is thus directly related to the total particle energy: $m\omega_B^2 Z_o^2/2$. In a Boltzmann distribution, we have

$$\langle J_o^2(k_{11}Z_o)\rangle = \frac{m\omega_B^2}{T_{oz}} \int_o^\infty J_o^2(k_{11}Z_o)\exp\left[-\left(\frac{m\omega_B^2 Z_o^2}{2T_{oz}}\right)\right] Z_o\, dZ_o$$

$$= \exp[-(k_{11}^2 T_{oz}/\omega_B^2 m)]\, I_o(k_{11}^2 T_{oz}/\omega_B^2 m)$$

$$\approx \frac{\omega_B}{k_{11}}\left(\frac{m}{2\pi T_{oz}}\right)^{1/2} \tag{16}$$

where $I_o(x)$ is the modified Bessel function, and T_{oz} is the initial parallel temperature.

With this result, (14) becomes

$$T_\perp(t) = T_{\perp o} + 1.58\ f_B \lambda T_{oz}^{-1/2}(k_{11}/k)P\ t^2 \ \ \text{keV} \tag{17}$$

where

f_B : bouncing frequency in MHz
λ : wave length of heating wave in meters
T_{zo} : parallel temperature in electron volts
P : heating wave power density in watts/100 cm^2
t : time is μs

For the second harmonic case ($n = 2$), the averaging of cosh bt is not easy. However because of the Jensen inequality, it can be shown that $\langle \cosh bt \rangle \geq \cosh[\langle b^2 \rangle^{1/2} t]$. Thus

$$T_\perp(t) \geq T_{\perp o} \cosh[\langle b^2 \rangle^{1/2} t] \tag{18}$$

$$\langle b^2 \rangle^{1/2} = 0.099 (k_\perp/k)(k_{11}/k)^{1/2} (f_B \lambda P)^{1/2} (T_{zo})^{-1/4} \ (\mu s)^{-1}$$

The units of various quantities are the same as those given in (17). The results show that for long interaction time, the second harmonic heating, which increases faster than an exponential increase in t, is more efficient than the fundamental, a similar conclusion to the previous results[10] obtained under the situation of exact resonance but without a bounce field. It thus implies that the bounce motion of electrons in the axial direction can indeed alleviate the detuning effect of frequency mismatch as mentioned before.

To study the nonlinear evolution of the excursion amplitude, Eq. (8) should be integrated. We first consider the case $|k_{11} Z| < 1$, and let $J_o(k_{11} Z) \simeq 1$ and $J_1(k_{11} Z) \simeq k_{11} Z/2$, thus Eq. (8) is solved to give

$$Z^2(t) = Z_o^2 \cos^2(|C_o| k_{11} t/2) \tag{19}$$

This result shows that, for electrons with initially small excursions, the excursion amplitude itself will oscillate slowly. For arbitrary values of the initial excursion Z_o, Eq. (8) is solved numerically and the results are shown in Fig. 1. Basically, for $k_{11} Z_o < 3.84$, the first finite zero of $J_1(x)$, the oscillatory behavior of $Z(t)$, Eq. (19), is seen. For larger values of Z_o, $k_{11} Z(t)$ decreases monotonically and approaches the next zero of $J_1(x)$

asymptotically. Since the successive zeros of $J_1(x)$ are separated by about π, the bounce amplitude of each particle is thus decreased by an amount less than $\lambda_{11}/2$, representing a small reduction of the parallel temperature.

The steady state bounce amplitude $Z(\infty)$ is seen to become discretized to the zeros of $J_1(x)$. For reasonably large λ_{11} and Z_o the discretization of $Z(\infty)$ will show up as heavy concentration of particles at the turning points where the instantaneous velocity is zero. This seems to be in agreement with experimental observation[15]. In the present model of a simple harmonic oscillation in z, it can be shown that the density is mildly singular as $(z - z_i)^{-1/2}$. However a slight spread of ω_B will destroy the singularity yielding a smooth density peak. For λ_{11} small and with an initally cool plasma, the separation between the discretized values of $Z(\infty)$ is small. A concentration of density at the mid-plane will then appear since all particles pass through the mid-plane and more particles have small excursions. Physically, this is due to the interaction of gyrating and bouncing electrons with the wave field. The phase relation between the motion and field evolves nonlinearly so that the axial secular force on the electrons point inward to the mid-plane.

IV. Discussion and Conclusion

Eqs. (17) and (18) represent rather efficient rates of heating. These results, although obtained through the approximation $J_n(x) \simeq (x/2)^n/n!$, are valid before reaching the relativistic regime. A discussion of the physics background is thus in order. In case of fundamental heating, we can express (12) on the simultaneous evolution of $v_\perp(t)$ and $\psi(t)$ by the triangular diagram of Fig. 2, where ϕ_o is determined entirely by the initial conditions of the particle. It is seen that all particles will gain in $v_\perp(t)$ eventually even for the most unfavorable initial condition $\cos\phi_o = -1$. $-\psi(t)$ evolves to approach ϕ_o asymptotically, in inphase with the wave field. For the case of the second harmonic heating, $n =$ we rewrite (13) as

$$v_\perp^2(t) = \frac{1}{2}v_{\perp o}^2[(1+\cos\phi_o)e^{bt} + (1-\cos\phi_o)e^{-bt}] \tag{20}$$

It is seen again that all particles, except the set of zero measure $\cos\phi_o = \pm 1$ (for $b \lessgtr o$ respectively), eventually will gain in $v_\perp(t)$. The effect of all particles gaining $v_\perp$ differs fundamentally from the stochastic approach where the particles diffuse in the $v_\perp - \psi$ plane. For fundamental heating, temperature increases in t to a power less than 2.

Finally, we would like to refer to the very important work by Jaeger et al[14] and by Lieberman and Lichtenberg[15] on ECH in a parabolic mirror. A comprehensive study was reported in the two papers cited which included, among other aspects of ECH, an extensive amount of numerical work from which most of quantitative conclusions were drawn. The detuning effect treated was that due to the inhomogeneous magnetic field of a mirror and the $k_{11}z$ effect was not considered. In spite of the difference, their numerical solution also indicates a t^2 growth of $\langle v_\perp^2 \rangle$. However, it is difficult to deduce from the paper the rate of heating for comparison. In contrast to the continuous interaction approach employed here, the impulse approximation was used. Whether continuous interaction can indeed predict similar improved heating rate in a mirror still remains an open question.

Since the background field configuration for practical devices is usually more complicated than the model used in the present analysis, only qualitative explanation to the experimental observation is expected. For a quantitative comparison, a more elaborate model should be adapted.

Acknowledgement

Research sponsored jointly by the Air Force Office of Scientific Research, Air Force Systems Command, USAF, under Grant Numbers AFOSR-79-7009 and AFOSR-83-0001, and by Fusion Energy Corp.

References

1. R.A. Dandl, F. Baity, K. Carpenter, J. Cobble, H. Eason, J. Glowienka, G. Haste, M. Hesse, S. Hiroe, N. Lazar, B. Quon, T. Uckan, T. White, Heavy Ion Beam Group, UV Spectroscopy Group, C. Hedrick, D. Batchelor, L. Deleano, R. Goldfinger, E. Jaeger, L. Owen, D. Spong, J. Tolliver, J. McBride, N. Krall and A. Sulton, Jr., Plasma Physics and Controlled Nuclear Fusion Research II, IAEA-CN-37/L-4, 365 (1979).

2. R. A. Dandl, H.O. Eason and H. Ikegam, Oak Ridge National Laboratory Report, TM-6703 (1979).

3. V.V. Alikaev, G.A. Bobrorskii, V.I. Poznyak, K.A. Razumova, V.V. Sannikov, Yu. A. Sokolov and A.A. Shmarin, Sov. J. Plasma Phys., 2. 212 (1976).

4. R.M. Gilgenbach, M.E. Read, K.E. Hackett, R. Lucey, B. Hui, V.L. Granatstein, K.R. Chu, A.C. England, C.M. Loring, D.C. Eldridge, H.C. Howe, A.G. Kulchar, E. Lazarus, M. Murakami, and J.B. Wilgen, Phys. Rev. Lett., 44, 647 (1980).

5. D.B. Batchelor, R.C. Goldfinger and H. Weitzner, IEEE Trans. on Plasma Science, Vol. PS-8, 78 (1980).

6. T.M. Antonson, Jr. and W.M. Manheimer, Phys. Fluids, 21, 2295 (1978).

7. H. Weitzner and D.B. Batchelor, Phys. Fluids 23, 1359 (1980).

8. O. Eldridge, Phys. Fluids 15, 676 (1972).

9. J.C. Sprott, Phys. Fluids 14, 1795 (1971); 15, 2247 (1972).

10. S.P. Kuo and B.R. Cheo, Phys. Fluids 24, 784 (1981).

11. J.Y. Hsu and S.C. Chiu, Phys. Rev. Lett. 45, 1561 (1980).

12. J.Y. Hsu, Phys. Fluids 25, 159 (1982).

13. J.P.M. Schmitt, Phys. Fluids 19, 245 (1976).

14. F. Jaeger, A.J. Lichtenberg and M.A. Lieberman, Plasma Phys. 14, 1073 (1972).

15. M.A. Lieberman and A.J. Lichtenberg, Plasma Phys. 15, 125 (1973).

16. B.H. Quon, R.A. Dandl, N.H. Lazar, and R.F. Wuerker, Bull. Amer. Phys. Soc. 26, 893 (1981).

FIGURE CAPTION:

Figure 1: Time Evolution of the Excursion Amplitude for cases of different initial values, where

$$|C_o| = (k_{\parallel}^2/k_{\perp} k)(cE/B_o)(\Omega/\omega_B)J_n(k_{\perp} v_{\perp o}/\Omega)\sin \phi_o.$$

Figure 2: Evolution of $v_{\perp}$-phasor during Fundamental heating.

Appendix

To derive the equations for the slow time varying functions $v_\perp(t)$, $\psi(t)$, $Z(t)$ and $\Phi(t)$, we substitute the resonance trajectory (5) into Eqs. (3) and (4). The integrand of the integral on the right hand side of Eq. (3) can be expressed by Bessel function expansion as

$$e^{i(\theta_o+\Omega t)}[(k_{11}\omega/k^2)-v_z]\cos(k_\perp x+k_{11}z-\omega t)$$

$$=\frac{1}{2}\sum_{\ell,\ell'}(k_{11}\omega/k^2-\ell'\ \omega_B/k_{11})J_{\ell'}(k_{11}Z)\Big\{J_{\ell-1}(k_\perp v_\perp/\Omega)$$

$$e^{i(\ell-n)(\theta_o+\Omega t)}\,e^{i[(\ell-1)\psi+\phi_o]}\,e^{i\ell'(\Phi+\omega_B t)}$$

$$+J_{\ell+1}(k_\perp v_\perp/\Omega)e^{-i(\ell-n)(\theta_o+\Omega t)}\,e^{-i[(\ell+1)\psi+\phi_o]}\,e^{-i\ell'(\Phi+\omega_B t)}\Big\} \qquad \text{(A1)}$$

where $\phi_o=k_\perp x_o-(k_\perp v_{\perp o}/\Omega)\sin\phi_o+n\theta_0$, $v_z=\omega_B Z\cos(\Phi+\omega_B t)$, $\omega=n\Omega$, and the expansion $e^{ia\sin\psi}=\sum_{P=-\infty}^{+\infty}J_P(a)e^{iP\psi}$ is used. Therefore, the slow time varying part of (A1) can be obtained by setting $\ell=n$ and $\ell'=o$, the result is

$$\Big\{e^{i(\theta_o+\Omega t)}[(k_{11}\omega/k^2)-v_z]\cos(k_\perp x+k_{11}z-\omega t)\Big\}_{\text{slow part}}$$

$$=(k_{11}\omega/k^2)J_o(k_{11}Z)\Big\{\frac{n\Omega}{k_\perp v_\perp}J_n(k_\perp v_\perp/\Omega)\cos(n\psi+\phi_o)$$

$$+i\,J_n'(k_\perp v_\perp/\Omega)\sin(n\psi+\phi_o)\Big\}e^{-i\psi} \qquad \text{(A2)}$$

Substituting (A2) into (3), yields

$$v_\perp e^{-i\psi} = v_{\perp o} + (k_{11}/k)(eE/m)\int_o^t dt' \; J_o(k_{11}Z')\Big\{(n\Omega/k_\perp v_\perp')J_n(k_\perp v_\perp'/\Omega)$$

$$\cos(n\psi' + \phi_o) + i \; J_n'(k_\perp v_\perp'/\Omega)\sin(n\psi' + \phi_o)\Big\} e^{-i\psi'} \tag{A3}$$

from which after first time derivative Equations (6) and (7) are obtained.

Similarly, the first term on the right hand side of Eq. (4) can be expanded as

$$[v_x - (k_\perp \omega/k^2)]\cos(k_\perp x + k_{11} \; z - \omega t)$$

$$= \frac{1}{2} \sum_{\ell,\ell'} [(\ell\Omega/k_\perp) - (k_\perp \omega/k^2)]J_\ell(k_\perp v_\perp/\Omega)J_{\ell'}(k_{11}Z)$$

$$\Big\{e^{i(\ell-n)(\theta_o + \Omega t)} e^{i(\ell\psi + \phi_o)} e^{i\ell'(\Phi + \omega_B t)} + \text{c.c}\Big\} \tag{A4}$$

Thus, the resonance term is obtained by keeping only terms in (A4) with $\ell = n$ and $\ell' = \pm 1$. The result is

$$\Big\{[v_x - (k_\perp\omega/k^2)]\cos(k_\perp x + k_{11} \; z - \omega t)\Big\}_{\text{resonance}}$$

$$= -2\,(k_{11}^2\omega/k_\perp k^2)J_n(k_\perp v_\perp/\Omega)J_1(k_{11}Z)\sin(n\psi + \phi_o)\sin(\Phi + \omega_B t) \tag{A5}$$

With the aid of (A5), Eq. (4) becomes

$$\frac{d^2}{dt^2} z + \omega_B^2 \; z = -2\,\omega_B \; C_o \; J_1(k_{11}Z)\sin(\Phi + \omega_B t) \tag{A6}$$

where $C_o = (k_{11}^2/k_\perp k)(cE/B_o)(\Omega/\omega_B)J_n(k_\perp v_{\perp o}/\Omega)\sin\phi_o$ and the invariant relation derived from Eqs. (6) and (7) has been used to reduce the expression of C_o. Eq. (A6) describes a harmonic oscillation driven by a resonant force, the solution of it is given by

$$z = z_o \cos \omega_B t + (v_{zo}/\omega_B)\sin \omega_B t + A(t)\cos(\Phi + \omega_B t) \tag{A7}$$

where $A(t) = C_o \int_o^t dt' \; J_o(k_{11}Z')$, and $|\dot{\Phi}| << \omega_B$ and $|\ddot{A}/A| << \omega_B^2$ are assumed.

From the original description $z = Z(t)\sin(\Phi + \omega_B t)$ and (A7), two self-consistent equations for $Z(t)$ and $\Phi(t)$ are obtained

$$\begin{aligned} z_o + \dot{A} \cos \Phi &= Z \sin \Phi \\ (v_{zo}/\omega_B) - A \sin \Phi &= Z \cos \Phi \end{aligned} \tag{A8}$$

Which are solved to obtain

$$Z^2 = z_o^2 + (v_{zo}^2/\omega_B^2) + A^2 + 2A[z_o \cos \Phi - (v_{zo}/\omega_B)\sin \Phi] = Z_o^2 - A^2 \tag{A9}$$

and

$$\Phi = \sin^{-1}\{[z_o Z + v_{zo}A/\omega_B]/Z_o^2 = \sin^{-1}\{[z_o Z \pm (v_{zo}/\omega_B)(Z_o^2 - Z^2)^{1/2}]/Z_o^2\} \tag{A10}$$

where the relation $z_o \cos \Phi - (v_{zo}/\omega_B)\sin\Phi = -A$ which can be derived by (A8) is used, $Z_o = (z_o^2 + v_{zo}^2/\omega_B^2)^{1/2}$, and $A = \pm(Z_o^2 - Z^2)^{1/2}$ is given by (A9).

Taking first time derivative on (A9), yields

$$\frac{d}{dt} Z = \mp C_o(Z_o^2 - Z^2)^{1/2} J_1(k_{11}Z)/Z \tag{A11}$$

The usage of plus or minus sign in (A10) and (A11) depends on the initial conditions. However, $|Z|$ is always decreasing from Z_o [i.e. $|Z| \leq Z_o$ as shown by (A9)], it is thus convenient to express (A11) and (A10) as Eqs. (8) and (9), respectively.

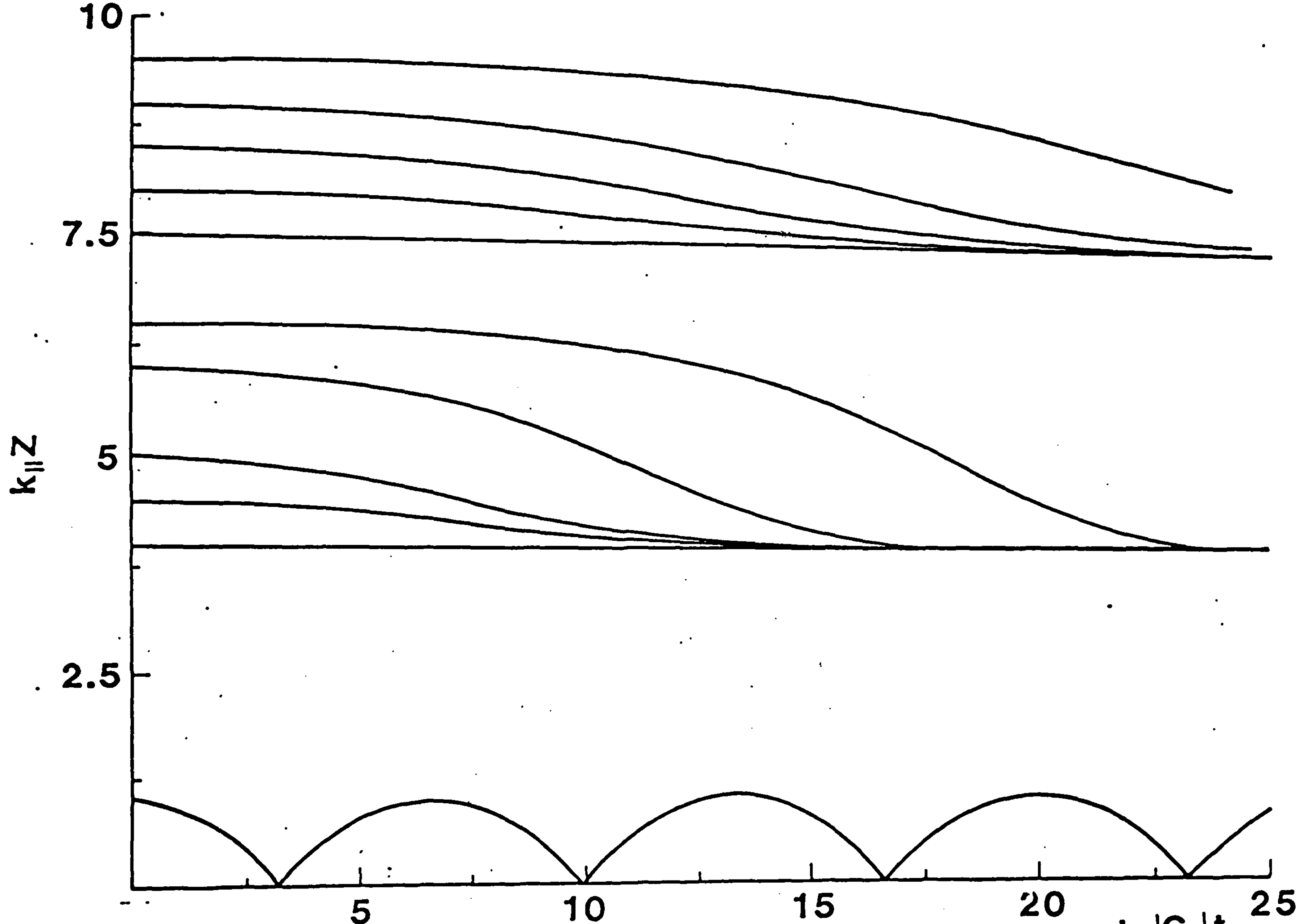

Fig. 1

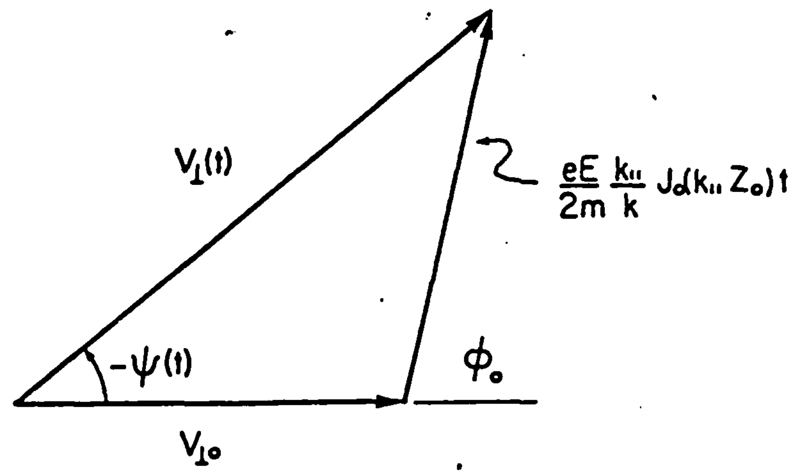
$V_\perp(t)$
$\frac{eE}{2m}\frac{k_\parallel}{k} J_\alpha(k_\parallel Z_0)t$
$-\psi(t)$
ϕ_0
$V_{\perp 0}$

Fig. 2

Turbulent heating of parametric instabilities in unmagnetized plasmas

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(Received 19 September 1980; accepted 11 March 1981)

Consideration of the effect of a uniform pump field on the particle orbits in a Vlasov plasma leads to a modified diffusion coefficient. When the particles oscillate in the pump field, the turbulent wave phase velocity seen by the particles is Doppler shifted by multiples of ω_0/k (ω_0 is the pump frequency). Hence, strong interactions between the particles and various components of the turbulent field will take place. It is shown that when the pump field is sufficiently strong: $E_0 \geq (4\pi n_0 T_e)^{1/2}$, bulk heating can dominate over tail heating and excitation of electrons to energy levels higher than the normal suprathermal values is possible. This field strength is within the range of laser fusion.

I. INTRODUCTION

During the past several years, substantial interest has been shown in the problem of anomalous heating[1-4] in a plasma driven by a large field oscillating near ω_{pe}. Parametric instabilities are excited and believed to be responsible for the observed heating of electron plasmas in laser fusion,[5] ionospheric modification,[6] computer simulation,[1,2,4] and laboratory experiments.[7,8] Heating of suprathermal tail electrons at about the phase velocity of the electron plasma wave is predicted in various theories.[2,4] Although the heating of suprathermal electrons is indeed observed in experiments,[7,8] bulk heating[2,7,8] and the production[4] of electrons with velocities several times the phase velocity of the electron plasma wave have also been observed in the laboratory experiments and computer simulations. Kaw *et al.*[9] analyzed the coupling between fast plasma waves and slow plasma waves through an independently excited ion density fluctuation. Bulk heating was also predicted. In this work, we have properly taken into account the pump field effect on the particle orbits, which has been neglected in previous theories. All wave components present are either parametrically excited instabilities or driven through wave-wave interactions. They possess wavelengths of about the same order. A modified diffusion coefficient is derived in Sec. II. This diffusion coefficient is used to study both bulk heating and tail heating. Substantial bulk heating and heating at higher than the suprathermal levels are predicted. The ratio of the energy deposition rates to the bulk electrons and the suprathermal tail electrons is estimated in Sec. III. A discussion is given in Sec. IV.

II. FORMULATION

We consider the problem of turbulent heating of plasmas by a strong electromagnetic pump wave through parametric excitation. The Vlasov equation can be expressed as

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_\alpha \cdot \frac{\partial}{\partial \mathbf{r}_\alpha} + \frac{q_\alpha}{m_\alpha}[\mathbf{E}_p(t) + \mathbf{E}(\mathbf{r}_\alpha, t)] \cdot \frac{\partial}{\partial \mathbf{v}_\alpha}\right) f_\alpha(\mathbf{v}_\alpha, \mathbf{r}_\alpha, t) = 0\,, \tag{1}$$

where α = electron, ion; $\mathbf{E}_p(t) = \mathbf{E}_0 \cos\omega_0 t$ is a prescribed spatially uniform pump field, and

$$\mathbf{E}(\mathbf{r}_\alpha, t) = \sum_{n,k} \mathbf{E}_{n,k} \exp\{i[\mathbf{k}\cdot\mathbf{r}_\alpha - (\omega + n\omega_0)t]\}$$

is the excited turbulent field with $\omega = \omega(k)$, the frequency of the excited ion wave. In most of the parametric theories,[10] only those waves with $n = 0, \pm 1$ which correspond to the ion wave and the up and down-shifted electron plasma waves, respectively, are considered. However, as the pump level is increased, higher frequency electron fluctuations $(m+1)\omega_0 \pm \omega$, $m = 1, 2, \ldots$, can also be excited through the coupling of the pump field and those fundamental instabilities ($n = 0, \pm 1$ modes). Since these higher frequency fluctuations are, in general, off-resonance modes, the thresholds for the parametric excitation of these higher-frequency fluctuations are high. Therefore, these fluctuations are not instabilities but driven waves sustained by the pump field and the fundamental instabilities. The relations between these higher frequency fluctuations and these fundamental instabilities are derived in the Appendix. Following the procedures by Dawson and Oberman,[11] and by Kuo and Cheo,[12] we now substitute the relations

$$\mathbf{v}_\alpha = \mathbf{v} + \frac{q_\alpha \mathbf{E}_0}{m_\alpha \omega_0} \sin\omega_0 t, \quad \mathbf{r}_\alpha = \mathbf{r} - \frac{q_\alpha \mathbf{E}_0}{m_\alpha \omega_0^2} \cos\omega_0 t \tag{2a}$$

into Eq. (1), and define

$$\begin{aligned}\epsilon(\mathbf{r}, t) &= \mathbf{E}[\mathbf{r} - (q_\alpha \mathbf{E}_0/m_\alpha\omega_0^2)\cos\omega_0 t, t] \\ &= \sum_{l,k} \epsilon_{l,k} \exp\{i[\mathbf{k}\cdot\mathbf{r} - (\omega + l\omega_0)t]\}\,,\end{aligned} \tag{2b}$$

where

$$\epsilon_{l,k} = \sum_n (-i)^{l-n} J_{l-n}(a_\alpha)\mathbf{E}_{n,k}\,, \quad a_\alpha = \frac{q_\alpha}{m_\alpha \omega_0^2}\mathbf{k}\cdot\mathbf{E}_0\,. \tag{3}$$

Thus, Eq. (1) becomes

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} + \frac{q_\alpha}{m_\alpha}\epsilon(\mathbf{r}, t)\cdot\frac{\partial}{\partial \mathbf{v}}\right) f(\mathbf{v}, \mathbf{r}, t) = 0\,, \tag{4}$$

where $f(\mathbf{v}, \mathbf{r}, t) \equiv f_\alpha[\mathbf{v} + (q_\alpha \mathbf{E}_0/m_\alpha\omega_0)\sin\omega_0 t, \mathbf{r} - (q_\alpha \mathbf{E}_0/m_\alpha\omega_0^2)\cos\omega_0 t, t]$, the distribution function in the $(\mathbf{r}, \mathbf{v})$ system.

We now proceed to derive the renormalization quasilinear diffusion equation. We notice that the pump field $\mathbf{E}_p$ is removed from the transformed Vlasov equation (4).

 0031-9171/81/061104-06$00.90

We assume that each component of the turbulent field $\boldsymbol{\epsilon}$ carries a random initial phase such that the ensemble average over the initial phases $\langle \epsilon_{l,k}\rangle = 0$ for all k and integers l. The distribution function is written as the sum of four parts as done by Dupree[13]

$$f = \langle f\rangle + f^{(c)} + \tilde{f} + f_m = \langle f\rangle + f^{(1)} + f_m , \tag{5}$$

where $f^{(1)} = f^{(c)} + \tilde{f}$, $\langle f\rangle$ is the average distribution function, $f^{(c)}$ is the phase coherent response to the electric field, $\tilde{f}$ describes the phase incoherent portion due to the nonlinear wave-wave interaction, and f_m describes all other effects which are to be ignored. Expanding the spatial dependence of $f^{(1)}$ in a Fourier series

$$f^{(1)}(\mathbf{v},\mathbf{r},t) = \sum_{l,k} f_{l,k} \exp\{i[\mathbf{k}\cdot\mathbf{r} - (\omega + l\omega_0)t]\}, \tag{6}$$

and substituting (2), (5), and (6) into (4), we obtain

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right)\langle f\rangle = -\frac{q_\alpha}{m_\alpha}\frac{\partial}{\partial \mathbf{v}}\cdot \sum_{l,k}\langle \epsilon^*_{l,k} f_{l,k}\rangle , \tag{7}$$

with $f_{l,k} = f^{(c)}_{l,k} + \tilde{f}_{l,k}$, where

$$f^{(c)}_{l,k} = -\frac{q_\alpha}{m_\alpha}\frac{\epsilon_{l,k}}{i[\mathbf{k}\cdot\mathbf{v} - (\omega + l\omega_0)]}\cdot\frac{\partial}{\partial \mathbf{v}}\langle f\rangle + \frac{q_\alpha^2}{m_\alpha^2}\sum_{k'\neq k}\frac{1}{i[\mathbf{k}\cdot\mathbf{v} - (\omega + l\omega_0)]}$$
$$\times \sum_n \frac{\partial}{\partial \mathbf{v}}\cdot \epsilon_{nk'}\epsilon^*_{nk'}\cdot\frac{1}{i[(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v} - (\omega - \omega' + (l-n)\omega_0)]}\cdot\frac{\partial}{\partial \mathbf{v}} f^{(c)}_{l,k}$$
$$+ \frac{q_\alpha^2}{m_\alpha^2}\sum_{k'\neq k}\frac{1}{i[\mathbf{k}\cdot\mathbf{v} - (\omega + l\omega_0)]}\sum_n \frac{\partial}{\partial \mathbf{v}}\cdot\epsilon_{nk'}\epsilon_{lk}\frac{1}{i[(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v} - (\omega - \omega' + (l-n)\omega_0)]}\cdot\frac{\partial}{\partial \mathbf{v}} f^{(c)*}_{nk'} , \tag{8}$$

and

$$\tilde{f}_{l,k} = \frac{q_\alpha^2}{m_\alpha^2}\sum_{k'\neq k}\frac{1}{i[\mathbf{k}\cdot\mathbf{v} - (\omega + l\omega_0)]}\sum_n\frac{\partial}{\partial \mathbf{v}}\cdot\epsilon_{n,k-k'}\epsilon_{l-n,k'}\frac{1}{i\{\mathbf{k}'\cdot\mathbf{v} - [\omega' + (l-n)\omega_0]\}}\cdot\frac{\partial}{\partial \mathbf{v}}\langle f\rangle + \dots . \tag{9}$$

Equation (8) can be solved iteratively, and $f^{(c)}_{l,k}$ can be expressed in terms of $\langle f\rangle$ including all the higher order terms. The result of the expansion can be obtained by solving an integro-differential equation for the averaged propagators in a way similar to that of Dyson.[14] We express approximately

$$f^{(c)}_{l,k} = -\frac{q_\alpha}{m_\alpha}\int_0^\infty d\tau \exp\{-i[\mathbf{k}\cdot\mathbf{v} - (\omega + l\omega_0)]\tau - \frac{1}{3}\mathbf{k}\cdot \mathrm{D}\cdot\mathbf{k}\tau^3\}$$
$$\times \epsilon_{l,k}\cdot\frac{\partial}{\partial \mathbf{v}}\langle f\rangle . \tag{10}$$

If only wave-particle resonant interactions are considered, we may substitute (10) into (7) and neglect the contributions from the nonlinear wave-wave interaction terms contained in (9). Thus, the renormalized quasi-linear diffusion equation is obtained as

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right)\langle f\rangle = \frac{\partial}{\partial \mathbf{v}}\cdot \mathrm{D}_\alpha\cdot\frac{\partial}{\partial \mathbf{v}}\langle f\rangle , \tag{11}$$

where

$$\mathrm{D}_\alpha = \frac{q_\alpha^2}{m_\alpha^2}\int_0^\infty d\tau \sum_{l,k}\langle \epsilon_{l,k}\epsilon^*_{l,k}\rangle$$
$$\times \exp\{-i[\mathbf{k}\cdot\mathbf{v} - (\omega + l\omega_0)]\tau - (\tfrac{1}{3})\mathbf{k}\cdot \mathrm{D}_\alpha\cdot\mathbf{k}\tau^3\} \tag{12}$$

is the renormalized diffusion tensor which includes resonance broadening due to the turbulent field ϵ and the effect due to the presence of the coherent pump field $\mathbf{E}_p$.

From Eq. (3), we write

$$\langle \epsilon_{l,k}\epsilon^*_{l,k}\rangle = \sum_{m,m'} (i)^{m-m'} J_{l-m}(a_\alpha)$$
$$\times J_{l-m'}(a_\alpha)\langle \mathbf{E}_{m,k}\mathbf{E}^*_{m',k}\rangle . \tag{13}$$

As shown in the appendix, waves at frequencies $(m+1)\omega_0 \pm \omega$, where $m = 1, 2, \dots$, are sustained by the coupling of the pump field and parametrically excited instabilities at frequencies ω and $\omega_0 \pm \omega$. Therefore, the ensemble average of (13) can be expressed in terms of the intensities of those instabilities ($n = 0, \pm 1$ components) as

$$\langle \epsilon_{lk}\epsilon^*_{lk}\rangle \approx \mathbf{k}\mathbf{k}\Big\{J_l^2(a_\alpha)|\phi_{0,k}|^2 + |\phi_{1,k}|^2\Big[J_{l-1}(a_\alpha) + \sum_{n=2}^{\infty} J_{l-n}(a_\alpha)\cdot\Big(1 - \frac{J_0^2(a_e)\omega_{pe}^2}{n^2\omega_0^2} + \frac{k_{de}^2}{k^2}J_n^2(a_e)\Big)^{-1}$$
$$\times \sum_{p=0}^{n-1}\frac{J_{-p}(a_e)J_{n-p-1}(a_e)}{(2n-p-1)^2}\frac{\omega_{pe}^2}{\omega_0^2}\Big]^2 + |\phi_{-1,k}|^2\Big[J_{l+1}(a_\alpha) + \sum_{n=-\infty}^{-2} J_{l-n}(a_\alpha)\Big(1 - \frac{J_0^2(a_e)\omega_{pe}^2}{n^2\omega_0^2} + \frac{k_{de}^2}{k^2}J_n^2(a_e)\Big)^{-1}$$
$$\times \sum_{p=n+1}^{0}\frac{J_{-p}(a_e)J_{n-p+1}(a_e)}{(2n-p+1)^2}\frac{\omega_{pe}^2}{\omega_0^2}\Big]^2\Big\} . \tag{14}$$

Substituting (14) into (12), the diffusion coefficient of the turbulent spectrum of the parametric instabilities is derived. This diffusion coefficient is different from the one derived in Refs. 15 and 16, in that the summations of the Bessel functions appear here. In the previously obtained diffusion coefficient, only tail heating by $|\phi_{\pm1,k}|^2$ is predicted. However, when this effect of the coherent pump field is included, additional coupling between waves and particles is also obtained. This can be observed from expression (14). For instance, the $l = 0$ term corresponds to the part of the diffusion coefficient for the bulk heating of electron plasma. This term includes contributions from both ion waves and electron waves. The $|l| \geq 1$ terms contribute to

heating of suprathermal electrons as shown in the work of Katz *et al.*[4] Also predicted here is the production of electrons of energies at higher than those of the suprathermal electron levels given by the $|l|=1$ terms. The production of those higher level electrons is also observed in the computer simulation work of Kruer and Dawson.[3] The additional coupling can be understood from the fact that particles are oscillating in the pump field; wave-particle interactions are thus Doppler shifted by the multiples of the oscillating frequency. This is similar to the higher harmonics in cyclotron resonances. If the resonance broadening effect is neglected in (12), the modified quasi-linear diffusion coefficient is then

$$D_\alpha = \frac{\pi q_\alpha^2}{m_\alpha^2} \sum_{l,k} \langle \epsilon_{l,k} \epsilon_{l,k}^* \rangle \delta[\mathbf{k}\cdot\mathbf{v} - (\omega + l\omega_0)]. \tag{15}$$

Again, we observe additional spikes of the diffusion coefficient in velocity space, not given in previous theories. If only up to the first order of the Bessel functions are kept, and if only the diffusion of electrons is of interest ($|a_i| \ll |a_e|$), (14) becomes

$$\langle \epsilon_{0,k}\epsilon_{0,k}^*\rangle \simeq \mathbf{kk}\{J_0^2(a_e)|\phi_{0,k}|^2 + J_1^2(a_e)(|\phi_{1,k}|^2 + |\phi_{-1,k}|^2)\}, \tag{16a}$$

$$\langle \epsilon_{1,k}\epsilon_{1,k}^*\rangle \simeq \mathbf{kk}\{J_1^2(a_e)|\phi_{0,k}|^2 + J_0^2(a_e)|\phi_{1,k}|^2\}, \tag{16b}$$

$$\langle \epsilon_{-1,k}\epsilon_{-1,k}^*\rangle \simeq \mathbf{kk}\{J_1^2(a_e)|\phi_{0,k}|^2 + J_0^2(a_e)|\phi_{-1,k}|^2\}, \tag{16c}$$

$$\langle \epsilon_{2,k}\epsilon_{2,k}^*\rangle = \mathbf{kk}\left[1 - \frac{5}{36}J_0^2(a_e)\left(1 - \frac{J_0^2(a_e)\omega_{pe}^2}{4\omega_0^2} + \frac{k_{de}^2}{k^2}J_2^2(a_e)\right)^{-1}\frac{\omega_{pe}^2}{\omega_0^2}\right]^2 J_1^2(a_e)|\phi_{1,k}|^2, \tag{16d}$$

$$\langle \epsilon_{-2,k}\epsilon_{-2,k}^*\rangle \simeq \mathbf{kk}\left[1 - \frac{5}{36}J_0^2(a_e)\left(1 - \frac{J_0^2(a_e)\omega_{pe}^2}{4\omega_0^2} + \frac{k_{de}^2}{k}J_2^2(a_e)\right)^{-1}\frac{\omega_{pe}^2}{\omega_0^2}\right]^2 J_1^2(a_e)|\phi_{-1,k}|^2. \tag{16e}$$

In reality, the exact diffusion coefficient given by (12) would include peaks at different points in velocity space each broadened to a finite interval around the peak. Similar to (15) these peaks are widely separated at a distance $|\omega_0/k|$ which has a value about several electron thermal speeds. Therefore, these intervals hardly overlap each other, and we may then decompose the diffusion coefficient D into

$$D = D_b + D_t,$$

where

$$D_b = \frac{e^2}{m^2}\int_0^\infty d\tau \sum_k \langle \epsilon_{0k}\epsilon_{0k}^*\rangle \times \exp[-i(\mathbf{k}\cdot\mathbf{v} - \omega)\tau - (\tfrac{1}{3})\mathbf{k}\cdot D_b\cdot\mathbf{k}\tau^3],$$

$$D_t = \sum_{n=1} D_{tn}, \tag{17}$$

and

$$D_{tn} = \frac{e^2}{m^2}\int_0^\infty d\tau \sum_k \sum_{l=\pm n} \langle \epsilon_{lk}\epsilon_{lk}^*\rangle \times \exp\{-i[\mathbf{k}\cdot\mathbf{v} - (\omega + l\omega_0)]\tau - (\tfrac{1}{3})\mathbf{k}\cdot D_{tn}\cdot\mathbf{k}\tau^3\}. \tag{18}$$

D_t and D_b are the diffusion coefficients for the suprathermal tail and for the bulk heating, respectively. The significance of D_b as shown in (17) is that $|\phi_{\pm 1,k}|^2$, the intensity of the electron plasma waves, which usually carries more energy, do contribute to bulk heating. No previous theories have demonstrated this effect. For oscillating two-stream instabilities, they are always the dominant terms for bulk heating. In decay instabilities, they become dominant when $J_1^2(a_e) > (\omega/\omega_0)J_0^2(a_e)$ as given by the Manley–Rowe relation. Moreover, $|\phi_{\pm 1,k}|^2$ also contributes to the production of higher energy electrons. This effect is included in Eqs. (16d) and (16e). Physically, the instability fields $\phi_{\pm 1,k}$ can again couple to the pump field in order to sustain fields oscillating at $2\omega_0 \pm \omega$ and thus transfer energy to electrons at the higher energy level. However, more important is that the time variation of the fields $\phi_{\pm 1,k}$ experienced by the oscillating electrons in the pump field is not at a single frequency as would be seen by a stationary observer. Instead, it contains a Doppler shifted spectrum. Thus, the resonant interaction between electrons with speeds at about $(2\omega_0 \pm \omega)/k$ range and $\phi_{\pm 1,k}$ also occur. Although contributions from these two mechanisms are out-of-phase, they never cancel each other. The Doppler shift effect always dominates. The arguments given are also true for the $|l|>2$ terms. Thus, significant heating and production of energetic electrons is also predicted as the pump power is increased. The heating rate and hence the ratio of the energy deposition rates to the bulk electrons and the suprathermal tail electrons will be analyzed in the next section.

III. ELECTRON HEATING RATE

In order to simplify the analysis, we only consider the one-dimensional case. In a homogeneous plasma, the spectra of parametric instabilities are symmetric in the dipole approximation, and hence the diffusion coefficient. Thus, the average background $\langle f\rangle$ in the oscillating frame is also symmetric in velocity space if the initial distribution is assumed to be a Maxwellian without drift. The zeroth moment of (11) gives the continuity equation, and the first moment of (11) indicates that there will be no drift velocity generated by this symmetric turbulent spectrum. The second moment of (11) thus determines the rate of energy gain due to the turbulent heating as

$$\frac{\partial}{\partial t}\epsilon = m\int \langle f\rangle \frac{\partial}{\partial v}[vD(v)]dv, \tag{19}$$

where $\epsilon = (m/2)\int v^2\langle f\rangle dv$. We now follow Dupree's argument,[15] the integrals of (17) and (18) may be approximated by step functions $U(x)$:

$$D_B(v) = D_0\left\{\left[U\left(v - \frac{\omega}{k_0} + W_B\right) - U\left(v - \frac{\omega}{k_0} - W_B\right)\right.\right. + \left.\left.\left[U\left(v + \frac{\omega}{k_0} + W_B\right) - U\left(v + \frac{\omega}{k_0} - W_B\right)\right]\right\}, \tag{20}$$

and

$$D_{Tn}(v) = D_n\left\{\left[U\left(v - \frac{n\omega_0}{k_0} + W_{Tn}\right) - U\left(v - \frac{n\omega_0}{k_0} - W_{Tn}\right)\right]\right.$$
$$\left. + \left[U\left(v + \frac{n\omega_0}{k_0} + W_{Tn}\right) - U\left(v + \frac{n\omega_0}{k_0} - W_{Tn}\right)\right]\right\}, \quad (21)$$

where

$$D_0 = \left(\frac{\pi e^2 3^{1/3}}{4m^2 k_0^{2/3}} \xi_{\omega 0}\right)^{3/4}, \quad D_n = \left(\frac{\pi e^2 3^{1/3}}{4m^2 k_0^{2/3}} \xi_{\omega n}\right)^{3/4},$$

$$\xi_{\omega 0} = \sum_k |\epsilon_{0,k}|^2, \quad \xi_{\omega n} = \sum_k (|\epsilon_{n,k}|^2 + |\epsilon_{-n,k}|^2),$$

$$W_B = (D_0/3k_0)^{1/3}, \quad W_{Tn} = (D_n/3k_0)^{1/3},$$

k_0 is the central wave number of the spectrum. Upon simplification of the diffusion coefficient, Eq. (19) becomes

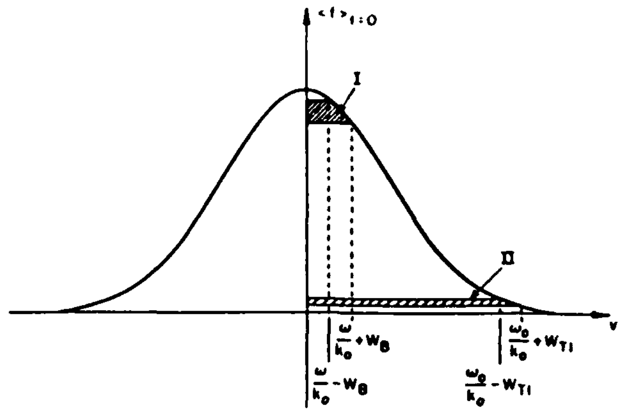

FIG. 1. Initial background distribution.

$$\frac{\partial}{\partial t}\epsilon = m\int \langle f\rangle D(v)dv + mD_0\left\{\left(\frac{\omega}{k_0} - W_B\right)\left[\left\langle f\left(\frac{\omega}{k_0} - W_B\right)\right\rangle + \left\langle f\left(-\frac{\omega}{k_0} + W_B\right)\right\rangle\right] - \left(\frac{\omega}{k_0} + W_B\right)\left[\left\langle f\left(\frac{\omega}{k_0} + W_B\right)\right\rangle + \left\langle f\left(-\frac{\omega}{k_0} - W_B\right)\right\rangle\right]\right\}$$
$$+ m\sum_{n=1}^{\infty} D_n\left\{\left(\frac{n\omega_0}{k_0} - W_{Tn}\right)\left[\left\langle f\left(\frac{n\omega_0}{k_0} - W_{Tn}\right)\right\rangle + \left\langle f\left(-\frac{n\omega_0}{k_0} + W_{Tn}\right)\right\rangle\right] - \left(\frac{n\omega_0}{k_0} + W_{Tn}\right)\left[\left\langle f\left(\frac{n\omega_0}{k_0} + W_{Tn}\right)\right\rangle + \left\langle f\left(-\frac{n\omega_0}{k_0} - W_{Tn}\right)\right\rangle\right]\right\}$$
$$= \frac{\partial}{\partial t}\epsilon_B + \frac{\partial}{\partial t}\epsilon_T, \quad (22)$$

where

$$\frac{\partial}{\partial t}\epsilon_B = m\int \langle f\rangle D_B(v)dv + 2mD_0\left[\left(\frac{\omega}{k_0} - W_B\right)\left\langle f\left(\frac{\omega}{k_0} - W_B\right)\right\rangle - \left(\frac{\omega}{k_0} + W_B\right)\left\langle f\left(\frac{\omega}{k_0} + W_B\right)\right\rangle\right], \quad (23)$$

and

$$\frac{\partial}{\partial t}\epsilon_T = m\int \langle f\rangle D_T(v)dv + 2m\sum_n D_n\left[\left(\frac{n\omega_0}{k_0} - W_{Tn}\right)\left\langle f\left(\frac{n\omega_0}{k_0} - W_{Tn}\right)\right\rangle - \left(\frac{n\omega_0}{k_0} + W_{Tn}\right)\left\langle f\left(\frac{n\omega_0}{k_0} + W_{Tn}\right)\right\rangle\right]. \quad (24)$$

By viewing Fig. 1, the right-hand sides of (23) and (24) are proportional to the areas of the shaded regions I and II, respectively. Hence, (23) and (24) may be approximated as

$$\frac{\partial}{\partial t}\epsilon_B \approx 2mD_0\left(\frac{\omega}{k_0}\left[\left\langle f\left(\frac{\omega}{k_0} - W_B\right)\right\rangle - \left\langle f\left(\frac{\omega}{k_0} + W_B\right)\right\rangle\right]\right.$$
$$+ W_B\left\{\langle f(0)\rangle - \tfrac{1}{2}\left[\left\langle f\left(\frac{\omega}{k_0} - W_B\right)\right\rangle\right.\right.$$
$$\left.\left.\left. + \left\langle f\left(\frac{\omega}{k_0} + W_B\right)\right\rangle\right]\right\} U\left(W_B - \frac{\omega}{k_0}\right)\right), \quad (25)$$

and

$$\frac{\partial}{\partial t}\epsilon_T \approx 2m\sum_n D_n \frac{n\omega_0}{k_0}\left[\left\langle f\left(\frac{n\omega_0}{k_0} - W_{Tn}\right)\right\rangle - \left\langle f\left(\frac{n\omega_0}{k_0} + W_{Tn}\right)\right\rangle\right]. \quad (26)$$

To calculate the ratio of the energy deposition rates into the bulk electrons and the suprathermal tail electrons, one has to derive $\langle f\rangle$ from Eq. (11). Although the diffusion coefficient is assumed to be constant in each heating region, as time evolves, the distribution function $\langle f\rangle$ will no longer be Maxwellian. The governing equation (11) may be solved numerically. However, we present a reasonable estimate of the distribution of the heating energy, by deriving the initial energy deposition rates in the following. We assume that $\langle f\rangle_{t=0}$ is Maxwellian. Equations (25) and (26) may be expressed explicitly to be

$$\frac{\partial}{\partial t}\epsilon_B\Big|_{t=0} \approx 2mD_0\left[-2W_B\frac{\omega}{k_0}\left\langle f'\left(\frac{\omega}{k_0}\right)\right\rangle\right.$$
$$\left. + W_B\{\langle f(0)\rangle - \tfrac{1}{2}[\langle f(-W_B)\rangle + \langle f(W_B)\rangle]\}\, U\left(W_B - \frac{\omega}{k_0}\right)\right]$$
$$\approx 2mD_0\left[2W_B\left(\frac{\omega}{k_0}\right)^2 \frac{m}{T_{e0}}\left\langle f\left(\frac{\omega}{k_0}\right)\right\rangle\right.$$
$$\left. + \frac{m}{2T_{e0}}W_B^3\langle f(0)\rangle\, U\left(W_B - \frac{\omega}{k_0}\right)\right], \quad (27)$$

and

$$\frac{\partial}{\partial t}\epsilon_T\Big|_{t=0} \approx 2mD_1\frac{\omega_0}{k_0}\left[-2W_{T1}\left\langle f'\left(\frac{\omega_0}{k_0}\right)\right\rangle\right]$$
$$\approx 4mD_1\left(\frac{\omega_0}{k_0}\right)^2 W_{T1}\frac{m}{T_{e0}}\left\langle f\left(\frac{\omega_0}{k_0}\right)\right\rangle, \quad (28)$$

where $\langle f\rangle = (m/2\pi T_{e0})^{1/2}\exp(-mv^2/2T_{e0})$ is assumed, and $n > 1$ terms in (26) are neglected. Therefore,

$$\left(\frac{\partial\epsilon_{B(0)}}{\partial t}\right)\left(\frac{\partial\epsilon_{T(0)}}{\partial t}\right)^{-1} \approx \left(\frac{J_1(a_e)}{J_0(a_e)}\right)^2\left\{\left[\omega^2\exp\left(\frac{-m\omega^2}{2k_0^2T_{e0}}\right)\right.\right.$$
$$\left.\left. + \frac{k_0^2W_B^2U(W_B - \omega/k_0)}{4}\right]\omega_0^{-2}\right\}\exp\left(\frac{m\omega_0^2}{2k_0^2T_{e0}}\right). \quad (29)$$

At moderate pump power levels, i.e.,

$$\eta_0 = E_0/(4\pi n_0 T_{e0})^{1/2} < 1, \quad |a_e| \ll 1,$$

and $J_1(a_e) \approx a_e/2$ and $J_0(a_e) \approx 1$. Thus,

$$k_0^2 W_B^2 = \left(\frac{k_0^2}{12}\frac{\pi e^2}{m^2}\xi_{\omega 0}\right)^{1/2} = \left(\frac{k_0^2}{48}\frac{\pi e^2}{m^2}a_e^2 2E_T^2\right)^{1/2}$$
$$\approx P^{1/2}(\pi/48)^{1/2}(k_0\eta_0\omega_{pe}/k_{de})^2, \tag{30}$$

where

$$\frac{2E_T^2}{8\pi} = \frac{1}{8\pi}\sum_k k^2(|\phi_{-1,k}|^2 + |\phi_{1,k}|^2) = \frac{PE_0^2}{8\pi}$$

is the total energy in all the plasma waves and $P = \gamma/\nu_{ei}$ is the factor of order one or larger by which the pump intensity exceeds its threshold value.[17] Substituting (30) into (29) we obtain

$$\left(\frac{\partial\epsilon_{B(0)}}{\partial t}\right)\left(\frac{\partial\epsilon_{T(0)}}{\partial t}\right)^{-1} = \left(\frac{k_0^2\eta_0^2}{4k_{de}^2}\right)\left\{\left[\omega^2 + P^{1/2}\left(\frac{\pi}{48}\right)^{1/2}\right.\right.$$
$$\left.\left.\times\left(\frac{k_0\eta_0\omega_{pe}}{16k_{de}}\right)^2 U(W_B - \omega/k_0)\right]\omega_0^{-2}\right\}\exp\left(\frac{m\omega_0^2}{2k_0^2T_{e0}}\right). \tag{31}$$

As an example we show that bulk heating can become equal to tail heating even at moderate pump power levels. We consider the case of oscillating two-stream instabilities: $\omega = 0$ and $\omega_0 \approx \omega_{pe}$. Using $k_0/k_{de} = \frac{1}{8}$, we obtain

$$\left(\frac{\partial\epsilon_{B(0)}}{\partial t}\right)\left(\frac{\partial}{\partial t}\epsilon_{T(0)}\right)^{-1} \approx 1,$$

when $\eta_0 \approx 0.187\ P^{-1/8} \leq 0.187$. Using $k_0/k_{de} = 0.2$, we obtain

$$\left(\frac{\partial\epsilon_{B(0)}}{\partial t}\right)\left(\frac{\partial}{\partial t}\epsilon_{T(0)}\right)^{-1} = 1,$$

when $\eta_0 = 0.62\ P^{-1/8} \leq 0.62$. However at

$$k_0/k_{de} = 0.25,\ \left(\frac{\partial\epsilon_{B(0)}}{\partial t}\right)\left(\frac{\partial}{\partial t}\epsilon_{T(0)}\right)^{-1} \approx 1,$$

when $\eta_0 \approx 1.52\ P^{-1/8} \leq 1.52$, this value is in the strong pump power region. Since strong Landau damping occurs in the region $k_0/k_{de} \geq 0.25$, instabilities have maximum growth rate in the region $k/k_{de} \leq 0.2$. Therefore, from the numerical values given here, it is reasonable to expect bulk heating to dominate over tail heating for a sufficiently strong pump: $|E_0| \geq (4\pi n_0 T_{e0})^{1/2}$.

IV. DISCUSSION

The perturbed orbit plasma-turbulence theories of Dupree[15] and of Weinstock[16] predict the broadening of the diffusion region as the turbulence level increases. Hence, the wave-particle interactions become more effective than the result of conventional quasi-linear diffusion. If the turbulence is parametrically excited by a coherent pump, the particle orbit is perturbed by the pump field in addition to the turbulent fields. The oscillation of the particle due to the pump field will also broaden the interaction between the turbulent fields. This is due to the fact that the phase velocity of the turbulent waves as viewed by the particles is Doppler shifted by integral multiples of ω_0/k, where ω_0 is the pump frequency and k is the wavenumber of the turbulent field. Therefore, the parametrically excited electron plasma waves do interact with those bulk electrons effectively. Moreover, the production of electrons at several phase speed of the electron plasma waves is also predicted. Numerical estimates show that bulk heating can dominate over tail heating for a sufficiently strong pump.

In the present analysis, it is seen that the value of $a_e = (k/k_{de})[E_0/(4\pi n_0 T_e)^{1/2}]$ is of critical importance to which part of the velocity spectrum be heated. At small values of a_e, bulk heating can be larger than tail heating provided that $E_0/(4\pi n_0 T_e)^{1/2} \geq 1$. Bulk heating is maximum at $a_e = 1.84$. At higher values of a_e the higher order Bessel functions can no longer be neglected and heating to much higher velocities than those of the suprathermal electrons are possible. For $E_0(4\pi n_0 T_e)^{-1/2} = 1$ the intensity of the pump is about 10^{16} W/cm² for glass laser and 10^{14} W/cm² for CO_2 laser ($T_e = 10$ keV). This is within the presently available intensities of both glass and CO_2 lasers.

ACKNOWLEDGMENT

This work was supported by the Air Force Office of Scientific Research, Grant AFOSR-79-0009.

APPENDIX

We now derive the relations between those higher frequency fluctuations $[(m+1)\omega_0 \pm \omega, m = 1, 2, \ldots]$ and the fundamental parametric instabilities $(\omega, \omega_0 \pm \omega)$ by using Vlasov's equation and Poisson's equation. The relations are only derived at the linear level, namely, we linearize Eq. (4); thus, the density perturbation δf_α in linear response to the total turbulent field ϵ in the oscillating frame is governed by

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right)\delta f_\alpha(\mathbf{v},\mathbf{r},t) = -\frac{q_\alpha}{m_\alpha}\epsilon(\mathbf{r},t)\cdot\frac{\partial}{\partial \mathbf{v}}f_{\alpha 0}(v), \tag{A1}$$

where $f_{\alpha 0}(v)$ is an unperturbed background distribution in the oscillating frame, it may be simply assumed to be Maxwellian. If we define an unperturbed trajectory: $(d/dt)\mathbf{r} = \mathbf{v}$ and $(d/dt)\mathbf{v} = 0$, and thus $\mathbf{v} = \mathbf{v}'$ and $\mathbf{r} - \mathbf{r}' = \mathbf{v}(t - t')$, where $\mathbf{r} = \mathbf{r}(t)$, $\mathbf{r}' = \mathbf{r}(t')$ and $\mathbf{v} = \mathbf{v}(t)$, $\mathbf{v}' = \mathbf{v}(t')$; hence, (A1) can be integrated and the result is

$$\delta f_\alpha(\mathbf{v},\mathbf{r},t) = -\frac{q_\alpha}{m_\alpha}\sum_{n,m,k}(-i)^m J_m(a_\alpha)$$
$$\times\exp\{i\mathbf{k}\cdot\mathbf{r} - i[\omega + (n+m)\omega_0]t\}$$
$$\times\frac{\mathbf{E}_{n,k}\cdot\partial/\partial\mathbf{v}f_{\alpha 0}}{i(\mathbf{k}\cdot\mathbf{v} - [\omega + (n+m)\omega_0])}. \tag{A2}$$

Since only electrostatic fluctuations are considered, we may define $\mathbf{E}_{n,k} = -ik\phi_{n,k}$ and substitute it and (A2) into Poisson's equation, which yields

$$\left[1 + \sum_\alpha\frac{k_{d\alpha}^2}{k^2}\sum_m J_m^2(a_\alpha)W\left(\frac{\omega + (n+m)\omega_0}{k(T_\alpha/m_\alpha)^{1/2}}\right)\right]\phi_{n,k}$$
$$= -\sum_\alpha\frac{k_{d\alpha}^2}{k^2}\sum_{\substack{l\neq n\\ m}}(i)^{l-n}J_{m-n}(a_\alpha)J_{m-l}(a_\alpha)$$
$$\times W\left(\frac{\omega + (n+m-l)\omega_0}{k(T_\alpha/m_\alpha)^{1/2}}\right)\phi_{l,k}, \tag{A3}$$

where

$$W(Z) = (2\pi)^{-1/2} \int_{-\infty}^{+\infty} \frac{x}{x - Z - i0^+} \exp\left(-\frac{x^2}{2}\right) dx$$

and

$$k_{d\alpha}^2 = 4\pi n_0 e^2 / T_\alpha \, .$$

Since those high-frequency waves are strongly off-resonance to the plasma modes, we may simply assume that they are sustained by the fundamental instabilities instead of being excited parametrically. Hence, they have a much smaller amplitude than the fundamental instabilities and (A3) may be approximated as

$$\phi_{n,k} \approx \left(1 - \frac{J_0^2(a_e)\omega_{pe}^2}{(\omega + n\omega_0)^2} + \frac{k_{de}^2}{k^2} J_n^2(a_e)\right)^{-1} (i)^{-(n+1)} \times \sum_{p=n+1}^{0} \frac{J_{-p}(a_e) J_{n-p+1}(a_e)}{(2n - p + 1)^2} \frac{\omega_{pe}^2}{\omega_0^2} \phi_{-1,k} \quad \text{for } n = -2, -3, \ldots ,$$

and

$$\phi_{n,k} \approx \left(1 - \frac{J_0^2(a_e)\omega_{pe}^2}{(\omega + n\omega_0)^2} + \frac{k_{de}^2}{k^2} J_n^2(a_e)\right)^{-1} (i)^{-(n-1)} \times \sum_{p=0}^{n-1} \frac{J_{-p}(a_e) J_{n-p-1}(a_e)}{(2n - p - 1)^2} \frac{\omega_{pe}^2}{\omega_0^2} \phi_{1,k} \quad \text{for } n = 2, 3, \ldots . \tag{A4}$$

With the aid of (A4) and (13), Eq. (14) is obtained.

[1] W. L. Kruer, P. K. Kaw, J. M. Dawson, and C. Oberman, Phys. Rev. Lett. 24, 987 (1970).
[2] W. L. Kruer and J. M. Dawson, Phys. Fluids 15, 446 (1972).
[3] P. K. Kaw and J. M. Dawson, Phys. Fluids 12, 2586 (1969).
[4] J. I. Katz, J. Weinstock, W. L. Kruer, J. S. DeGroot, and R. J. Faehl, Phys. Fluids 16, 1519 (1973).
[5] J. W. Shearer, S. W. Mead, J. Petruzzi, F. Rainer, J. E. Swain, and C. E. Violet, Phys. Rev. A 6, 764 (1972).
[6] H. C. Carlson, W. Gordon, and R. L. Showen, J. Geophys. Res. 77, 1242 (1972).
[7] K. Mizumo and J. S. DeGroot, Phys. Rev. Lett. 35, 219 (1975).
[8] M. Porkolab, V. Arunasalam, N. C. Luhmann, Jr., and J. P. M. Schmidt, Nucl. Fusion 16, 269 (1976).
[9] P. K. Kaw, A. T. Lin, and J. M. Dawson, Phys. Fluids 16, 1967 (1973).
[10] For example, K. Nishtkawa, J. Phys. Soc. Jpn. 24, 916 (1968).
[11] J. Dawson and C. Oberman, Phys. Fluids 5, 517 (1962).
[12] S. P. Kuo and B. R. Cheo, Phys. Fluids 21, 1753 (1978).
[13] T. H. Dupree, Phys. Fluids 15, 334 (1972).
[14] M. Porkolab and R. P. H. Chang, Rev. Mod. Phys. 50, 745 (1978), Sec. 5, p. 773.
[15] T. H. Dupree, Phys. Fluids 9, 1773 (1966).
[16] J. Weinstock, Phys. Fluids 12, 1045 (1969).
[17] W. L. Kruer in *Advances in Plasma Physics*, edited by A. Simon and W. B. Thompson (Wiley, New York, 1975), Vol. 6, p. 244.

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